# COLLAPSE OF THE WAVE FUNCTION, EINSTEIN-PODOLSKY-ROSEN PARADOX AND BELL's INEQUALITY 

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## 1. Collapse of the wave function

The large successes of quantum mechanics in explanation of a big part of physical phenomena leave not enough place for doubts in its regularity. Starting from the beginning of this century the quantum mechanics never came in a conflict with results of experiments and could make huge number of predictions.

Nevertheless, we should not forget about those many fundamental problems, which arose already at early stages of development of the theory. In that time the debates about these problems rendered large influence on the formation of quantum mechanics. But then the discussion moved to philosophical circles and has ceased to raise noticeable interest in physical circles. Among physicists the belief in the so-called Copenhagen or orthodox interpretation of quantum mechanics was ratified. This interpretation is connected, first of all, to the name of Bohr. Heisenberg, Dirac, Born and majority of other founders of quantum mechanics were supporters of this interpretation. The close position was shared by von Neumann.

At the same time, the actively working physicists did not usually go deep into arguing about the bases of quantum mechanics. Instead it the basic statements of the orthodox approach were frequently used as recipes. Two facts promoted that. At first, these recipes worked very well. Secondly, many basic statements were formulated in form of a series of principles: the principle of complementarity, the principle of indeterminacy, the projective principle. From the very beginning the physical content of these principles was not cleared up, rather it was offered to accept them in the form of a set of dogmas.

However, since the sixtieth years, interest to discussing the bases of quantum mechanics revives among physicists. Attempts, although not so successful, are undertaken to develop alternative approaches to quantum mechanics. In particular, various variants of the so-called ensemble approach begin to develop rather actively. Its supporters usually begin their genealogy from Einstein.

Kemble, Popper and Langevin were its supporters on early stage of development of this approach. In later time Everett, Ballentine, Bohm, Lande, Blokhintsev and many other worked in this direction.

The first separation of physicists into two large groups took place in connection with the problem of completeness of quantum mechanics as the physical theory. Bohr and other supporters of orthodox direction stated, that quantum mechanics gives complete description of an individual physical object, as much as it is possible. On the contrary, Einstein always thought, that quantum mechanics is a correct and complete statistical theory of ensembles, but not the theory of individual physical objects. Personally to me such point of view seems more convincing. Of course, the supporters of ensemble direction agree with Einstein.

The supporters of different directions differently estimate a role of wave function in quantum mechanics. In the orthodox interpretation the wave function is considered as attribute of an individual quantum object, in the ensemble interpretation it is considered as characteristic of an ensemble of identically prepared quantum systems.

The second point, on which there are the essential disagreements, is a problem of causality. In the orthodox interpretation the causeless phenomena are supposed. Most precisely this position is formulated in the famous book of von Neumann [1] about mathematical bases of quantum mechanics: " there is no experiment, which would support presence of causality, since the macroscopic experiments are as principle unsuitable for this purpose and quantum mechanics the only theory, which is compatible with the set of our experimental knowledge about elementary processes, contradicts it".

However such radical refusal from the principle of causality did not satisfy earlier and still does not satisfy many physicists. For example, without the principle of causality it is very complicated, if possible at all to solve the famous Einstein-Podolsky-Rosen paradox [2]. The solution of this problem, proposed by Bohr, does not seem to be very convincing. And apart from the Bohr solution a number of other variants were proposed within the framework of the orthodox interpretation. The large number of different variants indicates that none of them can satisfy majority of physicists. On the other hand, acceptance of the principle of causality allows to avoid this paradox.

In most variants of the ensemble approach the principle of causality is taken in the hard form. It is assumed, that there is some physical reality, which univalency determines the behaviour of any object. Though, for example, Blokhintsev adheres to other judgement. He understands the causality as some statistical regularity. Here his approach differs poorly from the orthodox one.

One of the most popular ways of introduction of a causality to quantum mechanics is the idea of hidden parameters, which claims that the state of an individual physical system is determined not only by a wave function, but also by some set of parameters, which are not taken into account in the apparatus of quantum mechanics.

Introductions of the hidden parameters makes many results of quantum mechanics easily interpretable, allows to solve series of paradoxes, which arise in the orthodox approach. However, as von Neumann [1] showed, the idea about hidden parameters contradicts the mathematical structure of the theory, accepted in quantum mechanics for description of observables and states.

True, Bell [3] showed, that in his proof von Neumann made the suppositions, which are too restrictive. They are really true for the standard apparatus of quantum mechanics, but in some its modifications, in particular in the model proposed by Bohm, these suppositions are not fulfilled. Therefore the program of hidden parameters cannot be considered as finally closed.

However simultaneously with partial rehabilitation of the hidden parameters Bell has deduced the famous inequalities, which should be fulfilled in models with the hidden parameters. These inequalities have two remarkable properties. At first, within the framework of standard quantum mechanics it is easy to specify such situation, when these inequalities are broken. Secondly, the inequalities can be checked experimentally. Apparently, these checks show, that the inequalities can be broken. True, the very persistent sceptics until now try to state, that the complete experimental proof of this fact is absent. But general scientific opinion is not on their side.

Among the modern supporters of the ensemble-causal approach in quantum mechanics so-called PIV-model is popular. This model is described rather explicitly in the recently published review of Home and Whitaker [4]. In this review the common comparative analysis of the orthodox and ensemble approaches is given.

The basic idea of the PIV-model consists in the supposition, that each physical system, being a member of quantum-mechanical ensemble, has " pre-assigned initial values " for all dynamical variables. On the one hand, these values distinguish different members of one ensemble. On the other hand, a part of these initial hidden values are made explicit by really performed measurements. The PIV-model has a lot of likenesses with the hidden parameters model. The PIV-model allows to solve quantum-mechanical paradoxes rather simply, but as far as I know, the problem of the Bell inequality is not solved in it.

Perhaps, it is the most difficult to believe in statement, which is done in the orthodox quantum mechanics about collapse of a vector of state. In the elementary form this statement is reduced to following. The state of a quantum-mechanical system is described by a vector of state $|\psi\rangle$. If definite basis is chosen in the space of states, this state can be described by the wave function $\psi$. By quantum-mechanical interactions the vector of state (the wave function) evolves in time, obeying the equation of motion, for example the Schrödinger equation. In this case the causality is observed in a complete volume. However such serene evolution is interrupted at a contact of the quantum object with a classical measuring device.

Let this measuring device measures an observable $Q$, which in the apparatus of quantum mechanics an operator $\hat{Q}$ corresponds to. Let $u_{i}$ being the eigenfunctions of the operator $\hat{Q}$ with eigenvalues $Q_{i}$

$$
\begin{equation*}
\hat{Q} u_{i}=Q_{i} u_{i} . \tag{1}
\end{equation*}
$$

Then in the measuring device, at first, expansion of wave function $\psi$ happens

$$
\begin{equation*}
\psi=\sum_{i} c_{i} u_{i} \quad c_{i}-\text { numbers }, \quad \sum_{i}\left|c_{i}\right|^{2}=1, \tag{2}
\end{equation*}
$$

secondly, with probability $\left|c_{k}\right|^{2}$ the wave function $\psi$ passes into one of the functions $u_{k}$

$$
\begin{equation*}
\psi \rightarrow u_{k} \tag{3}
\end{equation*}
$$

The transition (3) is just a collapse of wave function. This transition is purely probabilistic, the causal evolution is interrupted in it.

The statement about the collapse is formulated more accurately, if the statistical operator (density matrix) $\hat{\rho}$ is used instead of vectors of state. I shall remind, that to each pair of (normalized) vectors of stat $|\xi\rangle,|\eta\rangle$ there corresponds an operator

$$
\begin{equation*}
\hat{\rho}(\xi, \eta) \equiv|\xi\rangle\langle\eta| . \tag{4}
\end{equation*}
$$

The operator $\hat{\rho}(\xi, \xi)$ is a projector onto the state $|\xi\rangle$. Formulated by von Neumann in terms of operators $\hat{\rho}$ the statement about a collapse of wave functions looks as follows:

$$
\begin{equation*}
\hat{\rho}_{I}=|\psi\rangle\langle\psi| \equiv \sum_{i, j} c_{i} c_{j}^{*}\left|u_{i}\right\rangle\left\langle u_{j}\right| \rightarrow \hat{\rho}_{F}=\sum_{k}\left|c_{k}\right|^{2}\left|u_{k}\right\rangle\left\langle u_{k}\right| . \tag{5}
\end{equation*}
$$

The operators $\hat{\rho}_{I}$ and $\hat{\rho}_{F}$ describe a state of a quantum system before and after measurement, respectively. The operator $\hat{\rho}_{I}$ is the projector, in the sum over $i$ and $j$ to the left of the arrow nondiagonal $(i \neq j)$ terms are present, therefore the corresponding state is referred to as pure,. The operator $\hat{\rho}_{F}$ is equal to the sum of projectors, such state is referred to as mixed, in the sum to the right of the arrow the nondiagonal terms are absent. Transition from $\hat{\rho}_{I}$ to $\hat{\rho}_{F}$ is referred to as a projective principle.

Hereinafter the statement (5) was advanced by taking into account the state of the measuring device

$$
\hat{\rho}_{I} \otimes \hat{\sigma}_{I}^{A} \rightarrow \sum_{i, j} c_{i} c_{j}^{*} \mid u_{i}
$$

of the experiment in which the modification of wave function must be spread with a superlight velocity if we want to describe the results with the help of the collapse.

By the way, many modern physicists concern quite seriously a possibility to use a collapse for transmission of information with superlight velocity. The whole series of papers, published recently in UFN (see, for example [5]) is devoted to this problem. I consider such possibility as fantastic.

The orthodox quantum mechanics does not give the obvious physical mechanism of realization of collapse. Fuzzy reasonings on influence of a classical device and observer on quantum object are usually adduced. As it is usual, the most definite judgments can be found at the von Neumann's book [1]. He proposes to consider process of a measurement as some chain, which connects processes objectively flowing in nature with some " interior I " of the observer. Links of this chain are separate parts of the measuring device and parts of human body, ensuring process perceptions. In all these links the physical processes happen, obeying a principle of causality, but at some stage the physical reality transits by jump in mental perception. The causality is broken at such jump. We can move the jump along the chain at own discretion, but it necessarily should be somewhere.

Despite the whole ingenuity of these reasonings, they do not seem especially convincing. Besides interpretation of such toy situations as "Schrödingers's cat" or "Wigner's friends" are very difficult from such positions.

Of course, the attempts of interpretation of the collapse of wave function in purely physical terms were undertaken not once. There existed a popular explanation of the collapse by the influence on a quantum object of irreversible processes, which happen in the measuring equipment during the measurement. The supporter of such point of view was Rosenfeld. However it is very inconvenient to explain collapse from such positions in case of so-called negative experiment. In this experiment we conclude about properties of the quantum object not by means of registration of passage of this object through the device, but by means of registration of a lack of such passage. In this case the device should not influence the quantum object.

From my point of view, the scheme of description of collapse of wave function, developed at the moment by Namiki merits attention. This scheme is described in the review by Namiki and Pascazio [6] which is devoted to modern aspects of the quantum theory of measurements. The Namiki considers collapse not as a result of vanishing of any part of wave function, but as loss of coherence of various parts.

Schematically Namiki reasonings may be presented as follows. On the first stage (in the analyzer) the wave function $\psi$ of the quantum object is decomposed to components $u_{i}$ (the formula (2)) in the measuring device. On the second stage the detectors, included in structure of the measuring device, act on components $u_{i}$, chaotically changing their phase. Such deformed wave functions generate statistical operator, described by the left-hand side of the formula (5)

$$
\hat{\rho}_{I}=|\psi\rangle\langle\psi| \equiv \sum_{i, j} c_{i} c_{j}^{*}\left|u_{i}\right\rangle\left\langle u_{j}\right|,
$$

in which factors $c_{i}, c_{j}^{*}$ are gained with additional phases, depending on the number of the experiment. If after that we shall average the relation ( $5^{\prime}$ ) over many experiments, the nondiagonal elements are mutually compensated by randomness of phases, and the right-hand side of $\left(5^{\prime}\right)$ will pass into the right-hand side of (5):

$$
\hat{\rho}_{F}=\sum_{k}\left|c_{k}\right|^{2}\left|u_{k}\right\rangle\left\langle u_{k}\right| .
$$

The similar reasonings are carried out in the approach (see, for example [7] in which the supposition about collapse is substituted by the supposition about orthogonality of various states of the measuring device. Such scheme of reasonings is used in this approach. The average on many experiments can be described mathematically by evaluations of partial trace over the states of the device in the middle part of the formula (6)

$$
\sum_{i, j} c_{i} c_{j}^{*}\left|u_{i}\right\rangle\left\langle u_{j}\right| \otimes\left|W_{i}\right\rangle\left\langle W_{j}\right| .
$$

Due to orthogonality the trace of the nondiagonal operator $\left|W_{i}\right\rangle\left\langle W_{j}\right| \quad(i \neq j)$ is equal to zero, therefore the nondiagonal terms drop out in the formula ( $6^{\prime}$ ) and it will pass into the right-hand side of the formula (6).

However, as Namiki explains in his review, such vanishing of the nondiagonal terms is not equivalent to the collapse of wave function, since the coherence between various parts of the wave function is not broken. Therefore, if we shall put a synthesizer behind the detector, various parts of wave function will be coherently united in it. It should not happen at the collapse.

## 2. A quantum theory model with information field

Now I shall pass to the account of my version of the interpretation of quantum mechanics. The basic ideas of the version are published in papers $[8,9,10]$. However essentially modified variant will be explained here.

We shall assume, there is the complicated system of oscillations in the vacuum and they form a wave field. There are the local structureless singularities in the field. The quantum-mechanical elementary particle corresponds to each such singularity. This particle consists of singularity (nucleus of the particle) and regular part of the wave field, which is coherent to the nucleus of the particle. This part of the wave field forms shell of the particle.

The nucleus is a carrier of all observables, connected to the particle. These observables are stored in the latent form in the nucleus. Latter means, that not definite numerical magnitudes, but elements of some noncommutative algebra, correspond to observables. A part of these observables can pass from the latent form into explicit one, when the particle interacts with an approaching measuring device. It is supposed, the concrete result of measurement depends on state of the shell.

We shall assume that the state of the shell is unique for each particle. For example, it is possible to assume, the state of the shell depends on the whole previous
history of the particle, i.e. the shell is memory, which stores an information about a history of the particle. In this case the unrecurrence of state of the shell is easily explained by uniqueness of the history of each particle. One can find another consideration of histories in [11].

Uniqueness of state of each shell allows to reconcile two seemingly irreconcilable concepts - probability interpretation of quantum mechanics and hard determinism. Really, on the one hand, it is possible to think that the behaviour of each individual particle is univalently determined by its shell and state of the environment. It is stacked into the scheme of hard determinism. On the other hand, in a common case, the prediction for the behaviour of an individual particle can be only probabilistic, as any prediction is done on the basis of the previous experience. We fix, that definite consequences follow from definite set of causes. However, it is impossible univalently to predict consequences if this set of causes never repeats. At best, we can make a probabilistic prediction if it is possible to reveal some repeated subset of the causes.

Moreover, the state of shell cannot play a role of a hidden parameter, with the help of which the behaviour of the particle can be classified further, because each such class always consists of one element. In such situation a trivial statement is only possible: " the particle behaves, as it behaves ".

Thus, the corpuscular - wave dualism of quantum mechanics has the quite obvious interpretation in proposed model. The structureless local nucleus is a valence component of quantum particle, coherent oscillations of the nucleus and shell (extended in space) form wave information field of the particle. The information stored in this field determines a kind of mapping of latent values of observables in explicit values.

Collective oscillations of nuclei are possible in a complex, consisting of several particles. The vacuum oscillations, which are coherent to these collective oscillations, form an information field of the complex. This field plays role of multiparticle memory. The values of every possible correlation magnitudes are determined by a state of the multiparticle information field.

We shall discuss now, which place could be occupied by the quantum-mechanical wave function in the proposed model. It is clear, that the wave function should be tightly connected to information field, but they cannot coincide, as latter is unique, but the wave function describes the whole ensemble of quantum objects.

We shall introduce at first a concept of a global pure ensemble in this connection. It is supposed in proposed model, that the behaviour of a concrete quantum object is determined by information, stored in its information field. In order to use this information, at first we should reveal it. For this purpose the series of measurements should be performed. At each measurement the information field is subjected to influence of the measuring device, which changes an initial structure of the field. Fulfilling a sequence of measurements, we expose the information, concerning not one state of field, but to series of various states. Therefore only the part of the information can be detected for each state of the field .

We shall designate by an abbreviation MIES (maximal information exposed simultaneously) the information, stored in the information field, which can be detected as a result of a measurement of magnitudes which correspond to the mutually commuting elements of algebra of observables. Strictly speaking, the condition of simultaneity is not compulsory, but such abbreviation agrees well with the concept accepted in quantum mechanics - set of simultaneously measurable observables.

We shall name as a global pure ensemble a set of all quantum systems, at which the same MIES to be stored in memory. We shall consider, that such ensemble is described by an uniform wave function. The global pure ensemble is physically not realizable. In order to avoid possible complications we shall accept hypothesis, that any sufficiently large random sample of the global pure ensemble can serve as its rather good (in statistical sense) representative. This sample will refer to as a pure ensemble and will be characterized by the same wave function.

Mathematically we can determine wave function as an equivalence class of information fields of quantum objects, which store the same MIES in their memory. It is natural to assume, that the additive and wave properties of the wave function are consequences of corresponding properties of the information field. At the same time, the information field is not obliged to be the element of any Hilbert space as opposed to the wave function.

For the same quantum object the set of simultaneously measurable observables can be chosen by many various modes. Respectively, different variants the MIES can be extracted from memory of the same quantum object. Therefore the quantum object with fixed state of the information field can be referred to various pure ensembles.

Thus, the change of measuring devices, which are used for extraction of the information from memory of the quantum object, results in a modification of its wave function. This modification of wave function is not operated by any equation of motion. It has all indications of a collapse. If the information field of the quantum object does not change at such collapse, one can define this phenomenon as a passive or subjective collapse.

Besides, the quantum object can undergo a real physical action, which changes the information in its memory, but which is not described by quantum-mechanical equations of motion. Such modification of wave function is pertinent to define as an active or objective collapse.

For an illustration of this phenomenon we shall discuss experiment, the scheme of which is represented on Figure 1. The device consists of four mirrors (1,2,3,4) and three detectors $\left(D_{1}, D_{2}, D_{3}\right)$. The mirrors 1 and 4 are semipermeable. The detectors $D_{1}$ and $D_{3}$ are switched on coincidence, and the detector $D_{2}$ is switched on anticoincidence. Through the detector $D_{1}$ the photons are started into the device. The detectors $D_{1}$ and $D_{3}$ are necessary only for registration of photons. The detector $D_{2}$ plays central role in the phenomenon of collapse. At the device the photons either are reflected from mirrors, or pass through them. At reflection from mirrors the phase of oscillations changes on $\pi / 2$, at passage through a semipermeable mirror
the phase does not change. The mirror 1 plays role of the analyzer, in it the initial beam of photons is divided into two branches: 1-2-4 and 1-3-4. The mirror 4 plays two roles: synthesizer and second analyzer.


Figure 1.
We shall at first consider a variant of the experiment, when the detector $D_{2}$ is switched off (or is away). Elementary calculation of modification of phases of oscillations in the mirrors shows, that due to interference, the photons will not hit the detector $D_{3}$. Since the detectors $D_{1}$ and $D_{3}$ are switched on coincidence, the device will not register any positive result.

We shall now consider variant of the experiment, when the detector $D_{2}$ is switched on. Since the detector $D_{2}$ is switched on anticoincidence, only those events, when the photon does not pass through the detector $D_{2}$, are taken into account in the experiment. Therefore interferences will be away, in a mirror 4 and about the half of photons will hit the detector $D_{3}$. The result of experiment will be positive. This result completely agrees with the orthodox interpretation - due to a collapse the branch 1-2-4 of the wave function is destroyed.

In the Namiki interpretation this result is explained too. The branch 1-2-4 is not destroyed, but its phase chaotically move with respect by phases of the branch $1-3-4$. Therefore the interference effect disappears at averaging a number of events.

However now the experimental technique of weak beams has increased, so that experiments with separate photons are possible. It allows to check up presence of an interference (detector $D_{2}$ is switched off), when there is one photon in the device. The expected result of experiment is univalent (lack of a signal in the detector $D_{3}$ ) therefore averaging over a number of events is not required. The experiments with a similar scheme tvD6214(r)620621.5(e)0.1(n)28.7(t)6(s)-289.7(i)-2.4(s)-329.9(02(h)8.7(c)21TJfl $\Omega-1.465$ ?
ones. The dynamical interaction goes with participation of nucleus and results in a modification of dynamical characteristics of object. The information interaction can happen without participation of nucleus and, practically, without a modification of dynamical characteristics. For example, only the phases of oscillations change in wave field.

We shall suppose, that the information connection is strong inside quantum object so, if the phases of oscillations change in some region of an information field, this modification is quickly spread to the whole information field and the coherence of separate sections of this field is kept. However it will not happen if the sections are isolated from each other.

We shall return now to discussing the experiment. We shall at first consider a case, when the detector $D_{2}$ is switched off. The mirror 1 plays role of the analyzer and has two properties. At first, in it the shell of the photon is divided on two parts, which are hereinafter spread into two routes 1-2-4 and 1-3-4. The coherence of both parts is kept. Secondly, the mirror 1 is a point of a bifurcation for nucleus of photon. It signifies, that dynamically both routes are admissible for the nucleus. However due to information, stored in the shell, a choice of route will be made univalently. But from the point of view of quantum mechanics this choice will be random. The fact is, that the quantum mechanics deals not with information field, but only with its generalized performance - wave function. The various configurations of the information field correspond to the same wave function. On the other hand, with the help of preliminary measurements we can receive an information only about wave functions, therefore the choice of one of the routes by the nucleus will be random for us. Effectively the information connection between the nucleus and the shell plays role of random dynamical force in the point of bifurcation.

The phases of oscillations of the nucleus and separate sections of the information field can vary when they go on both routes, but the coherence of all components of the photon is kept. Therefore coherent synthesis (interference) of all constituents of the information field is possible in the mirror 4. Due to this interference the information field will not be spread toward the detector $D_{3}$. Therefore probability for the nucleus to hit the detector $D_{3}$ will be equal to zero.

We shall now consider the second variant of the experiment, when the detector $D_{2}$ is switched on. In a mirror 1 everything will happen the same way as in the first variant. Two scenarios are further possible, in which the nucleus will go by the route $1-3-4$, or will do by the route 1-2-4. In the scenarios with the route $1-3-4$ the nucleus hits the detector $D_{2}$. There the quantum object participates in dynamical and information interaction with the classical device. The device goes out of unstable equilibrium due to dynamical interaction with nucleus. The catastrophic process develops in the device. This process has microscopically observable result and the quantum object is registered.

Due to information interaction of quantum object with the detector the character of oscillations of nucleus and that part of the shell, which goes on route 1-3-4, changes. Since these components of quantum object strongly interact informational
among themselves, their coherence is kept. But the part of the shell, which goes by the route 1-2-4, appears to be isolated from this interaction. As a result, the wave field in the channel 1-2-4 loses coherence with the field in the channel 1-3-4. Since only the field, which is coherent with nucleus, is a part of quantum particle, the wave field of the channel 1-2-4 vanishes for the quantum object. Effectively the part of information, stored in the memory of the quantum object, is lost. This process of forgetting is irreversible.

This process results in a sharp modification of the information field of the quantum object. The wave function of the object also changes sharply thereof. This phenomenon has all features of objective (active) collapse. In spite of the fact that the wave function changes almost instantly in a large volume, any inconsistency with a relativity theory does not arise, as the wave field of the channel 1-2-4, which leaves structure of the quantum object, does not change. The modifications happen in the channel 1-3-4, i.e. the quantum object changes. Thus, the wave fields in the channels 1-2-4 and 1-3-4 do not disappear at the collapse, but these fields lose the coherence with each other. Therefore in the mirror 4 interferences will be absent.

We shall now address the second scenario, in which the nucleus goes by the route 1-2-4, and the nucleus-free wave field goes through the detector $D_{2}$. This field does not interact dynamically with the detector. In this case the cause, generating catastrophic process in the detector, is absent. Any microscopically observable of reaction of the device will not be present.

However the information interaction between the detector and wave field is available. This interaction will have no effect on behaviour of the detector, as it consists of huge number of incoherent microobjects. The additional random displacement of phases of oscillations will not change anything.

On the contrary, the information action of the detector will affect much the wave field in the channel 1-3-4. This field will lose coherence with the field and nucleus in the channel 1-2-4. The situation will be the same as in the first scenario. Thus the collapse develops in negative experiment in the same way as in the positive one.

## 3. EPR paradox and Bell's inequality

Perhaps, the most disputed situation arises in quantum mechanics at joint consideration of two problems: the Einstein-Podolsky-Rosen [2] paradox and the Bell inequality [3].

The explanation of the EPR paradox looks quite natural in the models with hidden parameters. Its explanation in the orthodox interpretation of quantum mechanics seem to me not so convincing. On the other hand, the Bell inequalities are easily proved in the models with hidden parameters. The experiment does not confirm these inequalities, and it agrees with the orthodox interpretation. I want to discuss, as these two problems are solved within the framework of the model proposed by me.

The joint consideration of the EPR paradox and the Bell inequalities is facilitated by that fact, that the same experiment is suitable for their illustration. Bohm [13] has proposed very obvious experiment for this purpose. It looks as follows (see Figure 2).


Figure 2.
A quantum object $Q$ (particle with spin 0 in the elementary variant of the experiment) decays into two objects A and B (particles with spins $1 / 2$ ). The objects A and B scatter at large distance and hit in measuring devices $\mathrm{D}(\mathrm{A})$ and $\mathrm{D}(\mathrm{B})$, respectively. The object A has a set of observables $A_{a}$ (double projection of spin onto the direction $a$ ), which differ by index $a$. The observables, corresponding to different indexes, are not simultaneously measurable. Each of the observables can take two values $\pm 1$. In a concrete measurement the device $\mathrm{D}(\mathrm{A})$ measures an observable $A_{a}$. For the object B everything is similar. In devices the $\mathrm{D}(\mathrm{A})$ and $\mathrm{D}(\mathrm{B})$ the measurements are independent.

We shall at first consider the Bell inequality. We shall assume, a quantum object Q has a hidden parameter $\lambda$. In each individual event the parameter $\lambda$ accepts a definite value. The distribution of events according to the parameter $\lambda$ is characterized by a measure $\mu(\lambda)$ with usual properties

$$
\mu(\lambda) \geq 0, \quad \int d \mu(\lambda)=1
$$

All magnitudes, concerning to an individual event, depend on the parameter $\lambda$. In particular, the values of observables $A_{a}$ and $B_{b}$, obtained in individual experiment, are functions $A_{a}(\lambda), B_{b}(\lambda)$ of the parameter $\lambda$. For the individual event the correlation of observables $A_{a}$ and $B_{b}$ is characterized by magnitude $A_{a}(\lambda) B_{b}(\lambda)$. The average value of this magnitude is referred to as correlation function $E(a, b)$,

$$
E(a, b)=\int d \mu(\lambda) A_{a}(\lambda) B_{b}(\lambda)
$$

Adding various values to indices $a$ and $b$ and taking into account that

$$
\begin{equation*}
A_{a}(\lambda)= \pm 1, \quad B_{b}(\lambda)= \pm 1 \tag{7}
\end{equation*}
$$

we shall receive an inequality

$$
\begin{gather*}
\left|E(a, b)-E\left(a, b^{\prime}\right)\right|+\left|E\left(a^{\prime}, b\right)+E\left(a^{\prime}, b^{\prime}\right)\right| \leq  \tag{8}\\
\leq \int d \mu(\lambda)\left[\left|A_{a}(\lambda)\right|\left|B_{b}(\lambda)-B_{b^{\prime}}(\lambda)\right|+\left|A_{a^{\prime}}(\lambda)\right|\left|B_{b}(\lambda)+B_{b^{\prime}}(\lambda)\right|\right]= \\
=\int d \mu(\lambda)\left[\left|B_{b}(\lambda)-B_{b^{\prime}}(\lambda)\right|+\left|B_{b}(\lambda)+B_{b^{\prime}}(\lambda)\right|\right] .
\end{gather*}
$$

In the right-hand side of the formula (8), due to the equalities (7), one of the expressions

$$
\begin{equation*}
\left|B_{b}(\lambda)-B_{b^{\prime}}(\lambda)\right|, \quad\left|B_{b}(\lambda)+B_{b^{\prime}}(\lambda)\right| \tag{9}
\end{equation*}
$$

is equal to zero, and other is equal to two for each value of $\lambda$. From here the Bell inequality follows

$$
\begin{equation*}
\left|E(a, b)-E\left(a, b^{\prime}\right)\right|+\left|E\left(a^{\prime}, b\right)+E\left(a^{\prime}, b^{\prime}\right)\right| \leq 2 . \tag{10}
\end{equation*}
$$

The correlation function $E(a, b)$ is easily calculated within the framework of standard quantum mechanics. In particular, when A and B are particles with spins $1 / 2$

$$
\begin{equation*}
E(a, b)=-\cos \theta_{a b}, \quad \theta_{a b} \quad \text { is an angle between a and } \mathrm{b} . \tag{11}
\end{equation*}
$$

It is easy to be convinced, that there are directions $a, b, a^{\prime}, b^{\prime}$, for which the formula (10) and (11) contradict each other.

However there are the conditions, when the inconsistency is absent. At deduction of the inequality (10) we silently supposed, that expressions (9) exist for each $\lambda$. But this supposition is erroneous, if the parameter $\lambda$ takes different values in each individual event. Really, the observables $B_{b}(\lambda), \quad B_{b^{\prime}}(\lambda)$ cannot have a definite value in one experiment, as they are not simultaneously measurable. On the other hand, the values of the parameter $\lambda$ in $B_{b}(\lambda), \quad B_{b^{\prime}}(\lambda)$ should be different in various experiments, as these values cannot repeat.

Just such situation is realized in the model proposed by me. In it the behaviour of an individual quantum object is determined by the information, which is stored in its information field. Formally this information can be considered as a hidden parameter. However each value of this parameter is unique by virtue of a unrecurrence of states of the information field. Thus, the given proof of the Bell inequality is not correct in the model proposed by me.

We shall consider now the EPR paradox. We shall carry out arguing on the basis of the same experiment. In brief, the paradox consists in the following. Let the device $\mathrm{D}(\mathrm{B})$ is located much further from the birthplace of the particles A and $B$ than the device $D(A)$. Let the device $D(A)$ measures a projection of spin onto the axis Z for the particle A, and this value appears equal $S_{z}(A)$. The particles A and $B$ were in a singlet state at once after decay of the object Q . Therefore the device D (B) will find out a value $S_{z}(B)=-S_{z}(A)$ for the particle B with probability equal to unity. Within the framework of the standard quantum mechanics it means that the particle B is in a state with definite value of projection of $\operatorname{spin}\left(-S_{z}(A)\right)$ on the axes Z.

We shall now assume, that we have thought better of it and have decided to measure projection of spin on the axes X for the particle $A$. Then repeating the previous reasonings, we shall receive, that the particle B will appear in state with definite value of projection $S_{x}(B)=-S_{x}(A)$. Thus, measuring the projection $S_{z}(A)$ or the projection $S_{x}(A)$ for the particle A, we place the remote particle B in one
or another state. It means, that the instant transmission of information on large distance exists.

Bohr proposed such an explanation to this paradox. We should not consider a dynamic system with correlation (pair of the particles A and B), as consisting of two separated and independent particles, but should treat the measurement on one particle as a measurement on the whole system. Einstein sharply objected to this reasoning since, in his opinion (and my too), it contradicts localization.

The paradox is absent in the model proposed here. The quantum object Q decays into two particles. The information fields of these particles keep correlation even after their separation from each other, as these particles have a common origin. Because of the correlation we can receive some information about structure of the information field of the particle B , producing a measurement on the particle A . It is a typical example of an indirect measurement, in which the structure of the information field of the particle B does not change, as the device $\mathrm{D}(\mathrm{A})$ has no long-range action.

Measuring the projection $S_{z}(A)$ for the particle A, we immediately expose MIES for the particle A and indirectly (by virtue of the correlation) we do MIES for the particle B . This indirect information is $S_{z}(B)=-S_{z}(A)$. If we measure the projection $S_{x}(A)$ by the device $\mathrm{D}(\mathrm{A})$, we can assign the particle B to pure ensemble, corresponding to the definite value $S_{x}(B)$. The transition in the device D (A) from one kind of measurement $\left(S_{z}(A)\right)$ to other type $\left(S_{x}(A)\right)$ is equivalent to change of a kind MIES, which we indirectly extract from the information field of the particle B, without changing the field. I name such phenomenon as a passive or subjective collapse.

We shall consider one more variant of experiment. Let the devices $D(A)$ and $\mathrm{D}(\mathrm{B})$ are at the same distance from the quantum object Q , and we simultaneously fulfill measurements of $S_{z}(A)$ and $S_{x}(B)$. As a result, we simultaneously obtain for the particle B the values of two observables $\left(S_{x}(B)\right.$ and $S_{z}(B)=-S_{z}(A)$ ), which are not simultaneously measurable in a usual terminology of quantum mechanics.

Such combination of direct and indirect measurements allows to receive a larger information, than MIES. Therefore, strictly speaking, the MIES is not a maximal information. However this information has a specific character - it corresponds to a physical state of quantum object in a definite time interval in the past. In the example considered by us this time interval is limited by moment $t_{0}$ (moment of decay of the object Q or birth of the particle B ) and moment $t_{1}$ (moment of measurement $S_{x}(B)$ in the device $\mathrm{D}(\mathrm{B})$ ). The indirect information, obtained by the device $\mathrm{D}(\mathrm{A})$, ceases to correspond to the state of the particle B after the moment $t_{1}$. Therefore such information is useless for predictions of a behaviour of the particle $B$ in future.

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