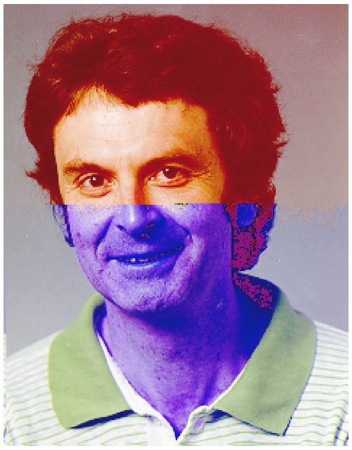

CLASSICAL ELECTRODYNAMICS: PROBLEMS OF RADIATION REACTION

A.A. Vlasov

Department of Physics, Moscow State University,
Moscow, 119899, Russia

There are known problems of Lorentz-Dirac equation for moving with acceleration charged particle in classical electrodynamics. The brief review of them is given. The question is formulated: can the superposition of retarded and advanced forms of self-interaction give possibility to solve problems of radiation reaction force. Some speculations in favour are considered.

0.

Since the famous Dirac's paper on relativistic radiation reaction in classical electrodynamics, many textbooks and research articles were published on that theme. Among them are [1-11], where one can find the discussion of the related problems: mass renormalization and its nonuniqueness, runaway solutions and the use of the advanced interaction.(These problems of radiation reaction one can find also in other classical field theories - scalar field theory, gravitational theory, etc.)

First of all we give brief review of some problems (sections 1-3) and then in section 4 consider the model of an extended particle.

1.

It is well known that the x-component of electric field $\vec{E}$ produced by point particle with charge $e = 1$ moving along x-axis is (Lienard-Wiechert solution; we take the units $c = 1$)

$$E = \frac{(1 + Nv^0)N}{(1 - Nv^0)(x - R^0)^2} \tag{1.1}$$

here $R^0 = R(t^0)$ - is the trajectory of particle motion and $v^0 = v(t^0) = dR^0/dt^0$ - is the particle velocity, t^0 - is the retarded time: $t^0 = t - |x - R^0|$, N - is the unit vector: $N = (x - R^0)/|x - R^0|$

To consider self-interaction one must expand field E near the particle. One can do it in different ways. For example: 1. Let $\epsilon = t - t^0$ (i.e. $|x - R^0| = \epsilon$), $\epsilon \to 0$. Then the expansion of all quantities in (1.1) in powers of ϵ gives for average value of E the following result

$$< E >= \frac{1}{2}\left[(E)_{N=1} + (E)_{N=-1}\right] =$$

$$\frac{2v}{\epsilon^2(1-v^2)} - \frac{2\dot{v}(1+v^2)}{\epsilon(1-v^2)^2} + \left[\frac{\ddot{v}(1+v^2)}{(1-v^2)^2} + \frac{(\dot{v})^2 2v(3+v^2)}{(1-v^2)^3}\right]. \tag{1.2}$$

2. Let $\epsilon = |x - R(t)|$, $\epsilon \to 0$. Then the similar calculations lead to

$$< E >= \frac{1}{2}\left[(E)_{N=1} + (E)_{N=-1}\right] =$$

$$-\frac{\dot{v}}{\epsilon} + \left[\frac{2\ddot{v}}{3(1-v^2)^2} + \frac{(\dot{v})^2 2v}{(1-v^2)^3}\right], \tag{1.3}$$

here dot means differentiation with respect to t, all quantities in $< E >$ are taken at moment t.

Infinite at $\epsilon \to 0$ terms in (1.2-3) lead to mass regularization in equation of particle motion. Finite at $\epsilon \to 0$ terms in (1.2-3) are what is usually called "the radiation force". Finite terms in (1.2-1.3) disagree each other and only finite term in (1.3) gives the known expression for Lorentz-Dirac radiation force.

Thus different ways of expansion of particle self-field may lead to different "radiation forces". From mathematical point of view it is obvious result - the expansion near infinity is not well defined operation.

This is a one of various aspects of the problem of mass regularization and its nonuniqueness (see also [5,17]).

2.

Among other problems, the one, which, to my opinion, draws not enough attention in the literature - is the problem of instability of preaccelerative "physical" solution of relativistic Lorentz-Dirac equation.

Let the point particle with mass m and charge e move under some external force F along x-axis. The relativistic Lorentz-Dirac equation reads:

$$\frac{m\dot{v}}{(1-(v/c)^2)^{3/2}} = F + \frac{2e^2}{3c^3}\left[\frac{\ddot{v}}{(1-(v/c)^2)^2} + \frac{3v(\dot{v})^2}{c^2(1-(v/c)^2)^3}\right], \tag{2.1}$$

here dot means differentiation with respect to $t,\ v = \dot{x}$.

Take the dimensionless variables η and τ, dimensionless force f and dimensionless radiation parameter γ and introduce scale multiplies a, b, p:

$$a \sim x_{cl},\ b \sim x_{cl},\ x_{cl} = \frac{e^2}{mc^2},\ p = \frac{a}{b}$$

$$x = a\eta,\ ct = b\tau,\ f = F\frac{b^2}{amc^2},\ \gamma = \frac{2}{3b}x_{cl},$$

here x_{cl} — the classical particle radius.

In terms of new variables equation (1) becomes

$$\frac{\ddot{\eta}}{(1-(p\dot{\eta})^2)^{3/2}} = f + \gamma\left[\frac{\dddot{\eta}}{(1-(p\dot{\eta})^2)^2} + \frac{3\dot{\eta}(\ddot{\eta})^2p^2}{(1-(p\dot{\eta})^2)^3}\right], \tag{2.2}$$

here dot means differentiation with respect to τ.

With the help of relativistic "velocity" u:

$$u = \frac{\dot{\eta}}{(1-(p\dot{\eta})^2)^{1/2}} = \frac{v/(cp)}{(1-(v/c)^2)^{1/2}} \tag{2.3}$$

"acceleration" w:

$$w = \frac{du}{d\tau} \tag{2.4}$$

and dimensionless proper time s:

$$\frac{b}{c(1-(v/c)^2)^{1/2}}\frac{d}{dt} = (1+(pu)^2)^{1/2}\frac{d}{d\tau} = \frac{d}{ds}$$

equation (2.2) can be put in more simple form:

$$w = f + \gamma\frac{dw}{ds}. \tag{2.5}$$

The "solution" of (2.5) is obvious:

$$w = w(s) = -\frac{1}{\gamma}\exp{(s/\gamma)}\int\limits_{s_0}^{s} dx f(x)\exp{(-x/\gamma)} \tag{2.6}$$

with "initial" value w_0:

$$w_0 = \left(\frac{\ddot{\eta}}{(1-(p\dot{\eta})^2)^{3/2}}\right)_{s=0} = \frac{1}{\gamma}\int\limits_{0}^{s_0} dx f(x)\exp{(-x/\gamma)}.$$

Integration of (2.4), taking into consideration (2.3) and (2.6), yields the following "solution" (strictly speaking, the integral equation) for particle velocity v:

$$\frac{1}{p}\ln\sqrt{\frac{(1+v/c)(1-v_0/c)}{(1-v/c)(1+v_0/c)}} =$$

$$\int\limits_0^s dz f(z) - \exp{(s/\gamma)}\int\limits_0^s dz f(z)\exp{(-z/\gamma)} + \gamma w_0\left(\exp{(s/\gamma)}-1\right), \tag{2.7}$$

here v_0- the "initial" velocity.

The form of Lorentz-Dirac equation similar to (2.7) was given in [2,3,4,7]. Our form (2.7) is convenient for analysis - from it immediately follows that:

(i) The peculiar features of Lorentz-Dirac equation do not qualitatively depend on scale multiplies a, b, p, so these features are valid as for "small", so for "large", "classical" distances.

(ii) If conditions of a problem permit to consider the limit $s \to \infty$ - all integrals in (2.7) are not divergent for $s < \infty$ (this condition is rather strong: it means, in particular, that the turning point of a particle trajectory, $v(s) = \dot{x} = 0$, must be integrable in the following sense: $\int ds f(s) = \int dx f(s(x))/\dot{x} < \infty$, and not all physically real trajectories do obey this condition), then all "solutions" (2.7) must be "runaway" - $|v| \to c$, with one exception.

(iii) This exception is the particular case of zero asymptotic value of "acceleration" w: $s_0 = \infty$ in (2.6) and

$$w_0 = \frac{1}{\gamma} \int\limits_0^1 dx f(x) \exp\left(-x/\gamma\right) \tag{2.8}$$

With (2.8), the R.H.S. of (2.7) takes the form

$$\int\limits_0^s dz f(z) + \exp\left(s/\gamma\right) \int\limits_s^1 dz f(z) \exp\left(-z/\gamma\right) - \int\limits_0^1 dx f(x) \exp\left(-x/\gamma\right) \tag{2.9}$$

Then for $s \to \infty$ and for "well defined" force f the velocity v does not reach c. But the price for this is the preacceleration and backward in time integration (see equation (2.9)) with accompanying paradoxes (some of them are discussed in [7,8,9]) (see also section 3.).

(iv) In literature the "solution" (2.9) is often called "physical", but from equation (2.7) it is easy to see that (2.9) is unstable under small deviations of "acceleration" w from zero value at infinite "future": due to (2.7) these initially small at $s = +\infty$ deviations δ grow at least as e^{δ}.

The instability of the "physical" solution one can also verify with the help of numerical calculation (see, for ex., [10]).

Following (iv) one can state that there are no stable "nonrunaway" solutions of Lorentz-Dirac equation, at least in one-dimensional case.

3.

Here we present one more paradox of preacceleration: the possibility of the formation of unavailable area of initial data, i.e. the formation of "event" horizon, absent in classical equation without radiation reaction.

Consider for simplicity the nonrelativistic case (with units $c = 1$).

Let the point particle with mass m and charge e move (following the classical nonrelativistic equation without radiation reaction) under the influence of the external force F along the x-axis:

$$\frac{d^2x}{dt^2} = \frac{F(x)}{m} \tag{3.1}$$

Let's take the force $F(x)$ in the form of a positive step (the form of the external force F is often used in the literature - for ex., [2,3])

$$F = \begin{pmatrix} 0, & x < -x_0 \\ mA & -x_0 < x < 0 \\ 0, & 0 < x \end{pmatrix} \tag{3.2}$$

here $A > 0, \quad x_0 > 0$. In the method of backward integration one must take the "final" data (i.e. for $t \to \infty$) and zero final acceleration. Thus for (3.2) we assume free motion of the particle in the future with velocity v:

$$x = vt. \tag{3.3}$$

Suppose that the point $x = 0$ is achieved at $t = 0$, and the point $x = -x_0$ - at $t = t_0 < 0$ (the value of t_0 we shall find below).

Then the integration of (3.1-2,3) with appropriate boundary conditions (for the second order equation (3.1) the position and the velocity of particle must be continuous) yields:

$$x = \begin{pmatrix} vt, & t > 0 \\ vt + At^2/2 & t_0 < t < 0 \\ ut + b, & -\infty < t < t_0 \end{pmatrix}. \tag{3.4}$$

Values of $u,\ b,\ t_0$ are determined from the matching conditions:

$$u = \sqrt{v^2 - 2x_0 A}, \quad t_0 = \frac{-v + u}{A}, \quad b = -x_0 - ut_0. \tag{3.5}$$

Following (3.4) in the regions free of force ($x > 0$ and $x < x_0$) the particle motion is free.

Consider the case of small velocity $\frac{dx}{dt} = u$ at the point $x = -x_0$:

$$u = \sqrt{v^2 - 2x_0 A} = \epsilon, \quad \epsilon \to 0. \tag{3.6}$$

We see that though the value of u is small, the particle can reach from the future all points on the x-axis: $-\infty < x < +\infty$.

Consider now the same particle with the same final data moving under the same external force F (3.2) but obeying the nonrelativistic Lorentz-Dirac equation

$$\frac{d^2x}{dt^2} - k\frac{d^3x}{dt^3} = \frac{F(x)}{m}, \tag{3.7}$$

here k ($k \approx x_{cl}$-the classical radius of a particle, $k > 0$) and the second term on L.H.S. of (3.7) deal with the radiation force.

As the equation (3.7) is of the third order, the acceleration of the particle also must be continuous. Then the solution of the above problem for equation (3.7) with appropriate boundary conditions yields (the point $x = 0$ is achieved at $t = 0$, and the point $x = -x_0$ - at $t = t_1 < 0$)

$$x = \begin{pmatrix} vt, & t > 0 \\ (v + kA)t + At^2/2 + k^2 A(1 - \exp(t/k)) & t_1 < t < 0 \\ ut + b + c\exp(t/k), & t < t_1 \end{pmatrix}. \tag{3.8}$$

From equation (3.9d) immediately follows that $c > 0$ for $t_1 < 0$.

Consider the case of zero particle velocity $\frac{dx}{dt}$ at the point $t = t_1 < 0, \quad x = -x_0$:

$$\frac{dx}{dt} = u + (c/k)\exp(t_1/k) = 0. \tag{3.10}$$

In (3.10) the value of u must be negative because the value of c is positive.

Equation (3.10) with the help of the system (3.9) can be rewritten as

$$z^2/2 - 1 - p = \exp(z)(z-1), \tag{3.11}$$

here $z = t_1/k < 0, \quad p = x_0/(k^2 A) > 0$. For our goal it is important to note that the equation (3.11) always has solution for $z < 0$ and this solution varies weakly with small changes in parameters of the problem under consideration.

Consequently if we consider the similar to (3.6) case of small particle velocity at the point $x = -x_0$:

$$\frac{dx}{dt}(t_2) = \epsilon, \quad \epsilon \to 0, \tag{3.12}$$

then $t_2 \approx t_1$ and, as for (3.10),

$$u < 0, \quad c > 0.$$

These inequalities lead to the conclusion that the particle velocity in the free of force region $(x < -x_0)$ - $\frac{dx}{dt} = u + (c/k)\exp(t/k)$, with positive value ϵ at $x = -x_0$, must inevitably take (with time decrease) zero value at some moment $t = t_{min} < t_2$ and some point $x = x_{min} < -x_0$.

So all x on the left of x_{min}: $x < x_{min}$, become unavailable on contrary to solution (3.6) of the equation (3.1), where all x-axis is available for the particle. Thus the event horizon is formed: not all initial data, physically possible from the point of view (3.1), can be achieved by backward in time integration method.

Of course, there can be large enough "exit" velocities v for which the horizon is not formed (i.e. for which u in (3.8) is positive: $u > 0$). But the main idea is that the horizon in the case of nonzero radiation reaction force *can* be formed contrary to the newtonian consideration without the radiation reaction force.

To illustrate this idea more explicitly, let's take the simple case of "pulse radiation", when

$$F/m = A\delta(x), \tag{3.13}$$

here $A > 0$ (see, for ex., [2,3])

Suppose that the point $x = 0$ is achieved at $t = 0$.

Then the matching conditions are:

$$x_+ = x_- = 0, \quad \dot{x}_+ = \dot{x}_-, \quad \ddot{x}_+ = \ddot{x}_- = -A/(kv).$$

The solution of the equation of motion (3.7) with F (3.13) is obvious:

$$x(t < 0) = -c_1 + (v - c_1/k)t + c_1 e^{t/k},$$

$$x(t > 0) = -c_2 + (v - c_2/k)t + c_2 e^{t/k},$$

$$c_2 = c_1 - Ak/v.$$

Absence of self-acceleration for $t > 0$ leads to the condition $c_2 = 0$, thus the trajectory is

$$x(t < 0) = Ak/v(e^{t/k} - 1) + (v - A/v)t,$$

$$x(t > 0) = vt.$$

We see that in the case of $v - A/v > 0$ the physics is standard. But if $v - A/v < 0$ (i.e. the exit velocity v is rather small) then the physics becomes nonstandard - there appears the horizon, i.e. appear points on the x-axis, unavailable from the future.

This paradox (as all the other) makes unrealizable the desire to solve problems of Lorentz-Dirac equation using preacceleration and backward in time integration.

4.

If to accept that there are problems with the Lorentz-Dirac equation, then one can try to solve them considering radiation forces of another type.

In the literature there are many examples of different forms of radiation forces. Let's mention some of them.

4.1

Following the works of Teitelboim school [5,6], when a charge is accelerated by a external field, it has a bound momentum P^i given by

$$P^i = P^i_{part} + P^i_{bound} = mu^i - m \cdot k \cdot \frac{du^i}{ds}, \quad k = 2e^2/(3m).$$

With it the Lorentz-Dirac eq. must be rewritten as

$$\frac{dP^i}{ds} = F^i + m \cdot ku^i \frac{du^n}{ds}\frac{du_n}{ds}.$$

4.2

In works of Plass [3] and Rohrlich [1] was shown that taking into account the asymptotic conditions (zero values for acceleration at infinity) the Lorentz-Dirac eq. can be rewritten in the form of integrodifferential (nonlocal) equation:

$$m\frac{du^i}{ds}(s) = \int\limits_0^1 d\alpha K^i(s + k \cdot \alpha)e^{-\alpha}$$

$$K^i = F^i - m \cdot ku^i u_n \frac{d^2u^n}{ds^2}.$$

4.3

One can rewrite the Lorentz-Dirac eq. using the iteration procedure considering the radiation term as small one (for recent paper see, for ex., [20]):

$$m\frac{du^i}{ds} = F^i;$$

$$m\frac{du^i}{ds} = F^i + m \cdot k(\frac{dF^i}{ds} - u^i u_n \frac{dF^n}{ds}) = F_1^i;$$

$$m\frac{du^i}{ds} = F^i + m \cdot k(\frac{dF_1^i}{ds} - u^i u_n \frac{dF_1^n}{ds}) = F_2^i$$

$$...etc$$

4.4

One can try to consider general form of the eq. of motion for a point particle using the distribution theory (Lozada, [14])

$$(\partial_i T^{ij}, \Phi) = 0$$

with T^{ij}- the total energy-momentum tensor for a point particle in an external electromagnetic field.

From this general eq. under some assumptions one can derive in particular the Bonnor's theory [21] with total nonconstant mass of a particle

$$m = m(s); \quad m\frac{du^i}{ds} = F^i,$$

$$\frac{dm}{ds} = m \cdot k\frac{du^n}{ds}\frac{du_n}{ds}.$$

4.5

One can draw the quantum field theory to these problems and to consider a quantum particle coupled to a quantum-mechanical heat bath (more, one can find the opinion in the literature that only quantum field approach can solve the problems of radiation reaction. I do no share this point of view as the only one possible). For such a system the description in terms of the quantum Langevin equation has a broad application (a far as I know, brownian model of synchrotronically radiating particle was first suggested by Sokolov and Ternov [18], for recent developments see , for ex., Ford et al [18]). Then in the nonrelativistic case the macroscopic eq., describing the particle motion, is

$$m\ddot{x} + \int_{-1}^{t} dt^0 \mu(t - t^0)\dot{x}(t^0) = f + F_{ext},$$

here μ - the memory function, f - a random force with mean zero.

For microscopic eq., see, for ex., Efremov [18].

All the above approaches use the retarded form of self-interaction. What new can give the consideration of a superposition of retarded and advanced self-interactions? For this sake let's take the model of extended (in some sense) particle.

4.6

Consider the hydrodynamic model of an extended particle [15].

Then the particle is described by the mass density $m \cdot f(t,x)$, charge density $\rho(t,x)$ and current density $j(t,x)$, obeying the continuity equations

$$m\frac{\partial f}{\partial t} + m\frac{\partial (v \cdot f)}{\partial x} = 0;$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial x} = 0 \tag{4.1}$$

here $v = v(t,x)$ is the hydrodynamic velocity of moving extended particle.

Let the particle move under the external force $F(t,x)$ along x-axis. Then the relativistic equation of its motion reads (we choose the units $c = 1$):

$$m\int dx f(t,x)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)u = \int dx \rho(t,x)E(t,x) + \int dx f(t,x)F(t,x) \tag{4.2}$$

here $v = v(t,x)$, $u = u(t,x) = v/\sqrt{1-v^2}$, $E(t,x)$ -is the electric field, produced by moving particle (the Lorentz force is absent in one-dimension case under consideration and internal forces give zero contribution to total force):

$$E = -\frac{\partial \phi}{\partial x} - \frac{\partial A}{\partial t} \tag{4.3}$$

and electromagnetic potentials ϕ and A are

$$\phi(t,x) = \int dx^0 dt^0 \frac{\rho(t^0,x^0)}{|x-x^0|}(a\delta_1 + b\delta_2),$$

$$A(t,x) = \int dx^0 dt^0 \frac{j(t^0,x^0)}{|x-x^0|}(a\delta_1 + b\delta_2), \tag{4.4}$$

with retarded and advanced delta-functions

$$\delta_1 = \delta(t^0 - t + |x - x^0|), \quad \delta_2 = \delta(t^0 - t - |x - x^0|)$$

and a, b - constants.

(The role of retarded and advanced interactions in electrodynamics is vividly described in the textbook [16].)

Substitution of (4.4) in (4.3) and integration by parts with the help of eq. (4.1) (taking zero values for integrals of exact integrands in x^0, i.e. $\int dx^0 \frac{\partial}{\partial x^0}(\rho \cdots) = 0$), yields

$$E(t,x) = \int \frac{dx^0 dt^0}{|x-x^0|^2}\left(\rho(t^0,x^0)\frac{x-x^0}{|x-x^0|}(a\delta_1 + b\delta_2) + j(t^0,x^0)(a\delta_1 - b\delta_2)\right). \tag{4.5}$$

Similar integration by parts for LHS of (4.2) gives the common result

$$LHS = \frac{dP}{dt}, \quad P = P(t) = m\int dx f(t,x)u(t,x), \tag{4.6}$$

here P - the particle momentum. Thus the eq. of motion reads

$$\frac{dP}{dt} = F_{self} + F_{ext},$$

$$F_{self} = \int dx\rho(t,x)E(t,x), \quad F_{ext} = \int dx f(t,x)F(t,x). \tag{4.7}$$

This eq. of motion has no second derivative of particle velocity; also there is no need in mass renormalization.

If the extended particle is compact, we can use in (4.5,7) the standard expansion in powers of $|x - x^0|$ (see, for ex.,[2]):

$$\delta(t^0 - t + \epsilon|x - x^0|) = \sum_{n=0}^{1} \frac{\epsilon^n |x - x^0|^n}{n!} \frac{\partial^n}{(\partial t^0)^n}\delta(t^0 - t)$$

with $\epsilon = \pm 1$.

Thus in nonrelativistic case we get the known result:

$$F_{self} = -(a+b)\int \frac{dx dx^0}{|x - x^0|}\rho(t,x^0)\rho(t,x)\frac{\partial v(t,x^0)}{\partial t} +$$

$$\frac{2}{3}(a-b)\int dx dx^0 \rho(t,x^0)\rho(t,x)\frac{\partial^2 v(t,x^0)}{(\partial t)^2}. \tag{4.8}$$

The first term in (4.8) is considered in literature as "-(electrodynamic field mass)×(acceleration)", and the second - as radiation reaction force.

The total change in particle momentum is

$$\Delta P = P(\infty) - P(-\infty) = \int dt \frac{dP}{dt}$$

and the change in particle momentum due to self-interaction is

$$\Delta P_{self} = \int dt F_{self}$$

Thus

$$\Delta P = \Delta P_{self} + \int dt F_{ext}. \tag{4.9}$$

Substitution of (4.5) into (4.7,9) gives

$$\frac{dP_{self}}{dt} = F_{self} = \int dt^0 dx dx^0 \frac{\rho(t,x)}{|x - x^0|^2}\cdot$$

$$\left(\rho(t^0,x^0)\frac{x - x^0}{|x - x^0|}(a\delta_1 + b\delta_2) + j(t^0,x^0)(a\delta_1 - b\delta_2)\right) \tag{4.10}$$

$$\Delta P_{self} = \int dt\, [RHS \;\; of \;\; (4.10)]\,. \tag{4.11}$$

The solution of (4.1) we can write as

$$\rho(t,x) = \frac{\partial \Phi(t,x)}{\partial x}, \quad j(t,x) = -\frac{\partial \Phi(t,x)}{\partial t}. \tag{4.12}$$

Then integration by parts in (4.10,11) with the help of (4.12) gives the following result:

$$\Delta P_{self} = \int dt dt^0 dx dx^0 \Phi(t,x)\Phi(t^0,x^0)\frac{x-x^0}{|x-x^0|^4}\cdot$$

$$\left[-6(a\delta_1 + b\delta_2) + 6|x-x^0|\left(a\frac{\partial \delta_1}{\partial t^0} + b\frac{\partial \delta_2}{\partial t}\right) - 2|x-x^0|^2\left(a\frac{\partial^2 \delta_1}{(\partial t^0)^2} + b\frac{\partial^2 \delta_2}{(\partial t)^2}\right)\right]. \tag{4.13}$$

In (4.13) the integrand is antisymmetric under transformations

$$t \to t^0, \quad t^0 \to t, \quad x \to x^0, \quad x^0 \to x$$

if

$$a = b \;\; (= 1/2).$$

Then the whole integral (4.13) has identically zero value:

$$\Delta P_{self} = 0.$$

So for an extended in one dimension particle the total change in particle momentum due to its self-interaction is zero (if is taken the half-sum of retarded and advanced interactions).

For $a = b$ the radiation term in F_{self} (4.8) is identically zero.

Similar result holds for energy balance: from equations (4.1,2) one can derive

$$\frac{d}{dt}W_{kin} = A_{self} + A_{ext} \tag{4.14}$$

with

$$W_{kin} = \int dx \frac{m \cdot f}{\sqrt{1-v^2}}$$

$$A_{self} = \int dx (j \cdot E), \quad A_{ext} = \int dx (F_{ext} \cdot v)$$

and

$$\Delta W_{kin} = \int dt \frac{d}{dt} W_{kin} = \int dt A_{self} + \int dt A_{kin}. \tag{4.15}$$

Substitution of (4.12) into (4.14) gives for $\int dt A_{self}$ identically zero result in the case

$$a = b = 1/2.$$

If $a \neq b$ then ΔP_{self} is not zero and more, the sign of ΔP_{self} is not identically negative - for some processes it can be negative and for the other - not. The latter case one can consider as "antidamping". (For example, if ρ and j have the form of moving "extended rigid" body with velocity $v(t) = dR(t)/dt$: $\rho(t,x) = A\exp[-\alpha(x - R(t))^2]$, $j = v(t)\rho(t,x)$, A, α-const, then for $a = 1$, $b = 0$ (retarded self-interaction) $\Delta P_{self} > 0$ if $v > 0$ for all t.)

Consequently we see that the problems of radiation reaction are connected not with the chosen form of radiation force but rather with the form of chosen self-interaction - retarded or/and advanced.

4.7

Let's postulate in section 4.6 the following relations

$$f(t,x) = \int dv f(t,x,v)$$

$$\rho(t,x) = Q \int dv f(t,x,v)$$

$$j(t,x) = Q \int dv v \cdot f(t,x,v) \tag{4.16}$$

with $f(t,x,v)$- the distribution function of extended in "v-dimension" particle. Thus we introduce the stochastic character of interaction of particle with its self-electromagnetic field (with "heat bath" in spirit of works [18, 19]). Distribution function $f(t,x,v)$ must obey the continuity equation:

$$\frac{\partial f(t,x,v)}{\partial t} + \frac{\partial(vf(t,x,v))}{\partial x} + \frac{\partial(\dot{v}f(t,x,v))}{\partial v} = 0.$$

With it the equations of sec.4.6, describing the system "particle + self-field", take the self-consistent nonlinear form [19] and the results of section 4.6 remain valid with the following interpretation:

the stochastic self-interaction of particle can give (with appropriate choice of superposition of retarded and advanced self-interaction) zero contribution to total changes in particle energy and momentum.

5.

In the literature one can often find the statement that the insert of E_{self} (or F_{self}, $W_{em,self} = \int dV(E^2 + H^2)_{self}/8\pi$, etc.) into equation of particle motion (4.2,7) is an obvious procedure justifying by the principle of extremum of action. Meanwhile the latter is formulated strictly only for closed systems or for systems, interacting with known external sources possessing known equations of motion. Systems with dissipation (with "damping" as a consequence of emission to infinity some amount of energy) may not obey the principle of extremum of action. Consequently the choice of the forms of E_{self} (or F_{self}, $W_{em,self}$, etc.), dealing with dissipation, can be considered as some supplementary hypothesis. Thus we can take for self-interaction the superposition of retarded and advanced forms of potentials (it should be stressed once more: we consider this superposition only for *self*-interaction and not for all solutions in the theory of electrodynamics).

- This is the "theoretical" point of view.

From "experimental" point of view we can stress the following.

Consider the equations (4.14) and (4.9) together with the balance equation of classical electrodynamics:

$$\frac{d}{dt}W_{em} = -I - A_{self} \tag{4.14}$$

where $W_{em} = \int dV(E^2 + H^2)/8\pi$ - the total energy (with the self-energy) in the volume V with surface S containing the moving source of electromagnetic field, and I - is the flux of Poynting-Umov vector through the surface S.

Then the latter can be put in the form

$_{tot}$

$$\frac{d}{dt} W_{tot} = -I + A_{ext}$$

with $W_{tot} = W_{em} + W_{kin}$ or

$$\Delta W_{tot} = -\int dtI + \int dtA_{ext}. \tag{5.1}$$

This equation is verified in experiments. More precisely, the R.H.S.'s of this equations - because th273.5mflΩ0.0DflΩlnhce ah273.5f

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