Composite Higgs Scalars in the Model of Dynamical Breaking of the Elektroweak Symmetry

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The problem of Higgs scalars are considered under assumption, that the scalars consists of the left doublet of the third generation and of the t-quark right singlet. The equation for an effective interaction vertex is considered and shown to have a solution. The use of this solution allows one to study the Bethe-Salpeter equation for $\bar{\psi}_L\, t_R$ bound state. The equation is shown to have two tachion solutions, which can be interpreted as Higgs scalars.

It is well-known, that the Standard Model (SM) of the electroweak interaction agrees excellently with the totality of experimental data. However, the Higgs scalar is not detected yet. On the other hand, the primordial elementary Higgs scalars cause some uneasiness in a formulation of SM. Indeed, the assumption of an existence of elementary scalars ϕ with Yukawa and $\lambda\,(\phi^+\phi)^2$ interaction leads to problems of triviality, of fine tuning for radiative correction in Higgs mass etc. In the present work we consider the variant of the dynamical electroweak symmetry breaking being proposed in [2, 3], which is connected with a selfconsistent mechanism of an appearance in the theory of the additional gauge-invariant vertex of electroweak vector bosons' interaction. This vertex effectively acts in the region of "small" momenta, restricted by a cut-off Λ being few TeV by the order of magnitude, which automatically appears in the theory. The vertex of interaction of W^+, W^-, W^0 with momenta and indices respectfully p, μ; q, ν; k, ρ has the form

$$\begin{aligned}
\Gamma(W^+,W^-,W^0)_{\mu\nu\rho}(p,q,k) \;&=\; \frac{i\lambda g}{M_W^2}\,F(p^2,q^2,k^2)\,\Gamma_{\mu\nu\rho}(p,q,k)\,;\\
\Gamma_{\mu\nu\rho}(p,q,k) \;&=\; g_{\mu\nu}(p_\rho(qk)-q_\rho(pk))+g_{\nu\rho}(q_\mu(pk)-k_\mu(pq))+\\
&+\; g_{\rho\mu}(k_\nu(pq)-p_\nu(qk))+k_\mu p_\nu q_\rho-q_\mu k_\nu p_\rho\,. \qquad (1)\\
F(p^2,q^2,k^2) \;&=\; \frac{\Lambda^6}{(\Lambda^2-p^2)(\Lambda^2-q^2)(\Lambda^2-k^2)}\,.
\end{aligned}$$

Here g is the gauge electroweak coupling constant, λ is the basic parameter of the model, nonzero value of which follows from the solution of a set of equations for parameters of the model [2]. This solution leads to masses of gauge bosons W and Z. We mean, that $W^0=\cos\theta_W\,Z+\sin\theta_W\,A$ is the neutral component of W triplet. Note, that anomalous vertices of the form (1) are often considered in the framework of a phenomenological analysis of possible deviations from the SM [4, 5].

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Equally with gauge bosons the t-quark has also a large mass. The importance of the t-quark for symmetry breaking was first emphasized in [6]. The origin of its mass in our approach is connected with an anomalous interaction of the t-quark [3, 7, 8, 9]. The possible deviations in $Z \to \bar{b}b$ decay and in A_{LR} are interpreted in the works being cited above. A brief review of the approach is presented in [10].

It is also very important for understanding of the structure of the theory to study scalar excitements in systems $W\,W$, $\bar{t}t$ etc., which could be interpreted as composite Higgs particles. In the present work we study a possible $(\bar{t},\,\bar{b})_L\,t_R$ excitations as a candidate to the Higgs scalar ϕ which has weak isotopic spin 1/2 as it is prescribed in SM. Note, that a virtual $W\,W$ variant leads rather to isotopic spins 1 and 0. It is evident, that a necessary tachion composite scalar needs a very strong interaction between the constituents, and the usual electroweak interaction is by no means sufficient for the goal. So we need an anomalous interaction and the model under discussion gives just such interaction for heavy quark generation.

So let us consider at first vertices interaction of $(t\,b)_L \equiv \Psi_L$ doublet and of $t_R \equiv \Psi_R$ singlet with gauge boson B of SM. We choose just B because the triplet W can not interact with t_R singlet. Now let us introduce new gauge invariant interactions

$$\Delta L_{int} \;=\; \xi_1\,\bar{\Psi}_L\gamma_\mu\Psi_L\,\partial_\nu\,B_{\mu\nu} \;+\; \xi_2\,\bar{\Psi}_R\gamma_\mu\Psi_R\,\partial_\nu\,B_{\mu\nu}\,, \tag{2}$$

where $B_{\mu\nu}$ is the corresponding field for B-boson. The corresponding vertices look like

$$V_{\mu,\,i}(k) \;=\; \xi_i\,(k^2\,\gamma_\mu \;-\; k_\mu\hat{k})\,\frac{1-(-1)^i\gamma_5}{2}. \tag{3}$$

Let us consider equations for vertices of LLB andRRB interactions. Denoting the corresponding function as Φ_i, $i\;=\;1,\,2$ and performing algebraic evaluations we have the following ladder equations

$$\Phi_i(p) \;=\; \xi_i \;+\; \frac{i\,\xi_i^2}{(2\pi)^4 3p^2}\int\frac{p^2q^2+3p^2(pq)-4(pq)^2}{(p-q)^2}\,\Phi_i(q)\,dq\,. \tag{4}$$

Here we choose the ladder summation for which the momentum of one spinor particle is set to zero and the momenta of the external B and Ψ are respectively $-\,p$ and p. In what follows we shall see, that just this kinematical region is appropriate for our task.

After the well-know procedure of Wick rotation and four-dimensional angular integration (see e.g. [11])

$$\Phi(x) \;=\; \xi \;-\; \frac{\xi^2}{96\,\pi^2}\int_0^\infty\Big(\frac{y(3x-2y)}{x^2}\theta(x-y)+\frac{x}{y}\theta(y-x)\Big)\,\Phi(y)\,y dy\,, \tag{5}$$

where index i is omitted and $x = -\,p^2,\; y = -\,q^2$ are corresponding momenta squared in the Euclidean space.

Using the standard tool of successive differentiation (see again [11]), we come to the following differential equation

$$\Big(\Big(x\frac{d}{dx}+2\Big)\Big(x\frac{d}{dx}+1\Big)\Big(x\frac{d}{dx}-1\Big) \;-\; \frac{\xi^2\,x^2}{16\,\pi^2}\Big)\,\Phi(x) \;=\; -\,2\,\xi\,; \tag{6}$$

with boundary conditions: $\Phi(0) < \infty$, $(x\,\Phi(x))|_\infty = 0$. After the substitution $x^2 = z$ we come to the well-known Mejer equation [12], and our boundary problem has the following solution

$$\Phi_i(x) = \xi_i\, F_i(x)\,;$$
$$F_i(x) = \frac{1}{2}\, G^{21}_{14}\Big(\beta_i\, x^2 \Big|^{0}_{1/2,\,0,\,-1/2,\,-1}\Big)\,; \qquad \beta_i = \frac{\xi_i^2}{128\pi^2}\,. \tag{7}$$

We have $\Phi_i(0) = \xi_i < \infty$, and the functions decrease at infinity as

$$\Phi_i(x) \simeq \frac{16\,\pi^2}{\xi_i^2\, x^2}\,. \tag{8}$$

Now we come to the conclusion, that the equations for the vertices under discussion have solutions provided there are primordial ξ_i. What about these quantities? Anomalous terms in $\bar t t$ and $\bar t b$ interactions [10] give rise to such terms. In this sense (and in the approximation being used) we have the necessary terms in the model. Here we would not fix values of ξ-s, preferring to consider them as free parameters.

Firstly we can obtain a restriction for ξ_1 from the decay $Z \to \bar b\, b$, because Z contain B with coefficient $\sin\theta_W$. Prescribing a deviation of the decay probability to this effect we obtain

$$\xi_1 = \frac{\Delta_b}{M_W^2}\, \frac{g\,((3-2\sin^2\theta_W)^2 + 4\sin^2\theta_W)\,\sqrt{\cot\theta_W}}{12\,(3-2\sin^2\theta_W)} = 0.26\,\frac{\Delta_b}{M_W^2}\,, \tag{9}$$

where $\Delta_b = 0.0035 \pm 0.0034$ [1] is a relative deviation of $Z \to \bar b b$ probability from that of SM. Of course, ξ_1 is consistent with zero.

Value (9) gives also forward-backward asymmetry of the decay A^b_{FB}. The maximal (1 s.d.) value of Δ_b gives $\Delta_{FB} = -0.005$ (see [8]) without any contradiction.

Estimate (9) allows to estimate an effective cut-off of the new interaction According to (8) we have

$$\Lambda_{eff} = \left(\frac{32\pi^2}{\xi_1^2}\right)^{1/4} \simeq 7.9\,TeV\,; \qquad for\ \Delta_b = 0.007\,. \tag{10}$$

This value turns to be of the order of magnitude of $\Lambda \simeq 5\,TeV$ in (1) being used in the model [10].

Let us now study Bethe-Salpeter equation for $\bar\Psi_L\, \Psi_R$ scalar bound state with interaction (2). Let $\bar\Psi_L$ momentum be $p + k/2$, Ψ_R momentum be $-\,p + k/2$ and a scalar bound state ϕ momentum be k. Then equation for $\phi(p,k)$ reads

$$\phi(p,k) = \frac{\xi_1\,\xi_2}{(2\pi)^4\, 4\, i} \int \frac{\gamma_\mu\, (\hat q + \hat k/2)\, \phi(q,k)\, (\hat q - \hat k/2)(t^2\gamma_\mu - t_\mu \hat t)}{(q^2 + k^2/4)^2 - (qk)^2}\, dq\,; \tag{11}$$
$$t = p - q\,.$$

Wave function has the following Lorentz structure

$$\phi(p,k) = X(p,k) + (\hat p \hat k - \hat k \hat p)\, Y(p,k)\,, \tag{12}$$

where X, Y are scalar functions. Substituting (12) into (11) and performing algebraic evaluation we obtain the following set of equations

$$X(p,k) \;=\; \frac{3\,\xi_1\,\xi_2}{16\,i\,(2\pi)^4}\int \frac{dq}{(q^2+k^2/4)^2-(qk)^2}\Bigl(X(q,k)\,(4p^2q^2-p^2k^2+4(q^2)^2-$$
$$-\,q^2k^2)\,+\,8\,Y(q,k)\,(p^2(qk)^2-p^2q^2k^2+q^2(qk)^2-(q^2)^2k^2)\Bigr)\,;$$
$$Y(p,k) \;=\; \frac{\xi_1\,\xi_2}{8\,i\,k^2\,(2\pi)^4)}\int \frac{dq}{(q^2+k^2/4)^2-(qk)^2}\Bigl(2\,X(q,k)\,(q^2k^2-(qk)^2)\,+ \qquad (13)$$
$$+\,Y(q,k)\,(4q^2-k^2)(q^2k^2-(qk)^2)\Bigr)\,.$$

We see from here, that $Y(p,k)$ does not depend on p and the dependence of $X(p,k)$ on p is quite simple, i.e.

$$X(p,k)\;=\;X(k^2)+p^2Z(k^2) \qquad Y(p,k)\;=\;Y(k^2)\,. \qquad (14)$$

Substituting (14) into (13) we see, that the momentum integral diverges. However, we have not taken into account our previous result (7) on formfactors of vertices. Indeed, in view of smallness of masses in comparison with effective cut-off, which are estimated by (10), we can consider momenta of legs to much less, than the integration momentum q. Then we have now just the same kinematic region, which we have studied above. Therefore, we use formfactors (7) and obtain instead of (13) after Wick rotation and angular integration

$$\begin{aligned}
X \;&=\; 6\,r\,\Bigl(I_{xx}\,X\,+\,I_{xz}\,Z\,+\,2\,I_{xy}\,Y\Bigr)\,;\\
Y \;&=\; -\,\frac{r}{2m}\Bigl(I_{yx}\,X\,+\,I_{yz}\,Z\,-\,2\,I_{yy}\,Y\Bigr)\,;\\
Z \;&=\; 6\,r\,\Bigl(I_{zx}\,X\,+\,I_{zz}\,Z\,+\,2\,I_{zy}\,Y\Bigr)\,;\\
r \;&=\; \frac{\xi_2}{\xi_1}\,;\qquad X\;\equiv\;X(k^2)\,;\quad etc\,.
\end{aligned} \qquad (15)$$

Here $m\;=\;k^2/4$ (Euclidean), that is $m>0$ means tachion mass of Higgs scalars $m_0\;=\;2\,\sqrt{m}$ and integrals are the following

$$\begin{aligned}
I_{xx} \;&=\; \int_0^\infty \frac{x-m}{x+m}\,F(x^2)\,F(r^2x^2)\,x^2\,g(x,m)\,dx\,;\\
I_{xy} \;&=\; \int_0^\infty h(x,m)\,F(x^2)\,F(r^2x^2)\,x^2\,dx\,;\\
I_{yx} \;&=\; \int h(x,m)\,F(x^2)\,F(r^2x^2)\,x\,dx\,;\\
I_{yy} \;&=\; \int g(x,m)\,F(x^2)\,F(r^2x^2)\,x\,(x-m)\,dx\,;\\
I_{zx} \;&=\; \int (1\,-\,h(x,m))\,F(x^2)\,F(r^2x^2)\,dx\,;\\
I_{xz} \;&=\; \int (1-h(x,m))\,F(x^2)\,F(r^2x^2)\,x^2\,dx\,;\\
I_{yz} \;&=\; I_{xy}\,;\quad I_{zy}\;=\;I_{yx}\,;\quad I_{zz}\;=\;I_{xx}\,;
\end{aligned} \qquad (16)$$

$$g(x,m) = \frac{\theta(m-x)}{m} + \frac{\theta(x-m)}{x};$$
$$h(x,m) = 1 - x\,\frac{x-m}{x+m}g(x,m)\,.$$

The calculations gives the result, that for $r < 1.2$ there is no solution of set (15, 16) and for $r > 1.2$ we have two tachion solutions and values of lower values m_0 in units $M = \sqrt{2}\,\Lambda_{eff}$ (see (10)) are presented at Table 1.

Table 1.

r	m	m_0/M
1.6	0.402	1.268
2.0	0.337	1.161
3.0	0.244	0.988
4.0	0.187	0.865
5.0	0.158	0.795
8.0	0.103	0.642
12.0	0.0712	0.534

In SM ratio of $\bar{t}_R t_R\, B$ and $\bar{\psi}_L \psi_L\, B$ interaction constants is

$$\frac{g_2}{g_1} = 4\,.$$

If one assumes, that for our parameter r this ratio is also valid, then $r = 4$ and two masses are

$$m_0(1) = 0.865\,M\,; \qquad m_0(2) = 2.387\,M\,. \tag{17}$$

Thus we come to the conclusion, that in the model under discussion there is a possibility to have composite Higgs scalars. The most important point consists in appearance of tachion solutions. The main qualitative features of the result:
1) The model favors two Higgs doublets – one "light" and another "heavy". (See other variants with two composite Higgses in [13].)
2) In the approximation used the mass of the "light" Higgs is of order of magnitude of few TeV.

The next step consists in calculation of effective $\phi\,\bar{\psi}_L\,t_R$ coupling and of constant of four-ϕ interaction. In this way one could construct an effective Higgs sector of our variant of the electroweak theory. At this step the theory will contain massive W, Z, t and one more or less "light" (few TeV) neutral Higgs scalar and four superheavy (few tens TeV) Higgses. All other particles are for the moment massless. However, CKM mixing leads to percolation of masses to other quarks. The farer from t are quarks the lighter are their masses. Such might be a qualitative picture of the model.

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