Determinism and Arrow of Time
Henri Poincaré Philosopher and Scientist

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Abstract

Even if today determinism is no more as essential as it has been in the development of science we must not forget that it was considered as the «condition sine qua non» of scientific facts all along the nineteenth century.

This guideline has been very useful, but it has also led to some blind alleys and Henri Poincaré was the first to understand that the scientists faced there a major problem. In the last years of his life he has tried to see further and to analyse the contradiction between determinism and free-will.

At the same time he developped the first elements of what is called today «chaotic motions, stange attractors, sensitivity to initial conditions ». He emphasizes that in unstable phenomena an exact prevision is impossible, that determinism looses there its physical meaning and that the statistical analysis becomes there more efficient.

These phenomena have a strong relation with the « arrow of time », it seems that chaotic motions are the true source of irreversibility of time and undeterminism. A simple example shows how these phenomena are interconnected.

Le déterminisme et la flèche du temps. Henri Poincaré philosophe et savant

Résumé

Même si aujourd'hui le déterminisme n'a plus l'importance qu'il a eu autrefois dans l'histoire de la science, il ne faut pas oublier qu'au dix-neuvième siècle il était considéré comme la condition « sine qua non » des faits scientifiques.

Cette idée a été très utile pour mettre de l'ordre dans la multitude des faits observés, mais elle a aussi conduit à un certain nombre d'impasses et Henri Poincaré fut le premier à comprendre que les scientifiques butaient là sur une question majeure. Dans les dernières années de sa vie il a tenté de voir plus loin et d 'analyser la contradiction entre le déterminisme et le libre-arbitre.

A la même époque il développa les premiers éléments de ce qui est appelé aujourd'hui « mouvements chaotiques, attracteurs étranges, sensibilité aux conditions initiales ». Il souligna que dans les phénomènes instables une prévision rigoureuse est impossible, qu'à ce niveau le déterminisme perd sa signification physique et que les analyses statistiques deviennent alors plus efficaces.

Ces phénomènes sont très liés avec ce que l'on appelle la « flèche du temps », il semble que les mouvements chaotiques soient la source véritable de l'irréversibilité du temps et de l'indéterminisme. Un exemple simple aide à comprendre les liens de toutes ces questions.

1. The creed of scientism

At the end of the nineteenth century, the impressive progress of Science have led to an entirely new situation. Most scientists, but also many writers and philosophers as well as a very large proportion of the public feeled that mankind was at the dawn of a new era.

Science was considered as almost infallible, as able to solve all problems, worries and miseries that were the age-old share of human condition, as able to answer to all questions especially the philosophical ones: where are we? where do we come from? where do we go? why are we on Earth?

This particularly optimistic spirit was reflected into what can be called the creed of scientism :

1) Science will explain everything.
2) Religions belong to the past (Auguste Comte).
3) All that actually exists can be proved (I only believe what I can see).
4) God is an invention of men (Freud, Feuerbach).
5) The Universe is infinite and unchanging, it has always existed, it will exist forever.
6) Man is an animal, that is some organized matter.
7) Evolution only depends of random (Darwin).
8) The Bible, the miracles are only legends (Renan)
9) The finality is only an appearance, only the determinism actually exists.

Of course the philosophy corresponding to this creed is the materialism and the determinism, and the corresponding belief is atheism. But even in the vicinity of 1900 this creed was impossible to accept fully and the German biologist and physiologist Ernst-Wilhelm von Brücke (1819-1892) has claimed: « The finality is an exacting mistress and a biologist cannot avoid her, but above all he refuses to be seen publicly with her ! ».

We must notice that , even if this creed has met many difficulties, contradictions and refutations all along the twentieth century, it remains for many scientists the inconscious, but nevertheless very active, basis of their vision of science and of their definition of scientific facts. Furthermore many laws of modern nations reflect this philosophy of determinism, and murderers are sometimes considered as not guilty : are they not predetermined ?

Today we know that the 1900 creed of science has always less and less grounds. The scientists have met many limits of science, the most famous being the following :

The uncertainty principle (Heisenberg).

The Gödel theorem of incompleteness.

The chaotic motions, the strange attractors, the sensitivity to initials conditions (Henry Poincaré, Gaston Julia, Benoît Mandelbrot, Michel Hénon).

The butterfly effect (E. N. Lorenz).

The Liapunov time, the time of divergence (Ruelle, Takens, Bergé, Lighthill).

The paradox of freedom.

The limits of information theory.

Even in astronomy, this stronghold of determinism, the time of divergence of motions is not infinite; it is about 10 to 100 millions of years in the solar system and Celestial Mechanics cannot decide alone of the origin of the Moon or of the long-term evolution of planets.

On the other hand, as recognized by Robert Oppenheimer in a dramatic statement, «the scientists have met sin», firstly the chemists with the war of asphyxiating gas (1915-1918) and the industrial death of nazis camps, secondly the physicists with the atom bomb and the nuclear accidents of Tchernobyl type and thirdly the biologists with the temptation of eugenics, the genetic manipulations and the experiments on aborted human foetus collected alive at the gates of hospitals... The image of science at the service of mankind has gone.

As a result most scientists are now modest ; they know that Science cannot, by far, explain everything. They have created many ethical commitees and have looked outside of Science for directives and justifications. They have recognized, after René Cassin, that the main references of the human conditions, such as the Rights of Man, have an ethical and religious origin : the belief in the dignity of human beings.

2. Henri Poincaré philosopher

Henri Poincaré has written many books at the boundary of Science and Philosophy such as *«La Science et l'hypothèse»* (Science and Hypothesis), *«La valeur de la Science»* (Science's value), *« Science et méthode »* (Science and method). But we will here only consider his reflexions on determinism and irreversibility as they appear in his last and unfinished book that has been entitled *«Dernières pensées »* (Last thoughts).

In the relations between Science and Ethics, Henri Poincaré recognizes many benefic effects : The scientists are looking for truth, their ethics leads them to be honest and to have a general point of view leading usually to the good of all mankind. However he was distressed by the philosophical problem of determinism :

«Mais nous sommes en présence d'un fait; la science, à tort ou à raison, est déterministe; partout où elle pénètre elle fait entrer le déterminisme. Tant qu'il ne s'agit que de physique ou même de biologie, cela importe peu; le domaine de la conscience demeure inviolé; qu'arrivera-t-il le jour où la morale deviendra à son tour objet de science? Elle s'imprégnera nécessairement de déterminisme et ce sera sans doute sa ruine»[1].

«However we are in the presence of the following fact : truly or wrongly Science is deterministic, its extension is also an extension of determinism. As long as only Physics or even Biology are concerned the effects are minor; but what will happen when Ethics will become a subject of science? It will be impregnated with determinism and will probably be destroyed »[1].

We can almost write that Henri Poincaré was already horrified by the future horrors of the reign of such a dogmatic Science and of ‘’scientifically founded regimes’’ that send you to the gulag archipelago not because of your crimes but because of your social origins... (today such a policy is qualified as « crime against Mankind »).

3. Henri Poincaré scientist: the determinism

What is determinism? it is the following idea : «Two experiments with exactly the same initial and limit conditions must give exactly the same results ». It is easy to understand how precious this idea has been in the development of science and in the observation, analysis and classification of the innumerable phenomena of nature.

Celestial Mechanics is the best example of the application of determinism. The marvellous law of universal attraction was sufficiently simple to be discovered by Newton’s genius and sufficiently complex to give a wide variety of motions with many perturbations and *inequalities*. It was above all a deterministic law leading to an accurate prediction of planetary motions and eclipses. These success were the major reason of the consensus of nineteenth century scientists about determinism and the discovery of planet Neptune after the long calculations of Leverrier and Adams was of course an excellent positive argument.

With such a background it was easy to imagine that all the phenomena of nature were deterministic and this was formalized by Laplace in1814 in his book *Essai philosophique sur les probabilités :*

«Nous devons envisager l’état présent de l’Univers comme l’effet de son état antérieur et la cause de ce qui va suivre. Une intelligence qui, pour un instant donné, connaîtrait toutes les forces dont la nature est animée et la situation respective des êtres qui la composent, si d’ailleurs elle était assez vaste pour soumettre ces données à l’analyse, embrasserait dans la même formule le mouvement des plus grands corps de l’Univers et ceux du plus léger atome; rien ne serait incertain pour elle, et l’avenir comme le passé seraient présents à ses yeux »[2].

« We must consider the present state of Universe as the effect of its past state and the cause of its future state. An intelligence that would know all forces of nature and the respective situation of all its elements, if furthermore it was large enough to be able to analyze all these data, would embrace in the same expression the motions of the largest bodies of Universe as well as those of the slightest atoms. All futur and all past would be as known as present for this intelligence »[2].

This absolute determinism is known as the Laplacean determinism, it reflects perfectly the conditions of the development of science: It was easier to study first the most simple, regular and foreseeable phenomena such as the rise of the Sun, the periodic recurrence of full Moon, of seasons, of high tides etc. and a natural generalization has led to consider that all natural phenomena must be deterministic.

We have seen that Henri Poincaré was distressed by this conclusion and he proceeded to a careful analysis. He noticed first that we must make a clear distinction between what can be called « mathematical determinism » and « physical determinism ». The mathematical determinism reflects the definition: « Two experiments with exactly the same initial and limit conditions must give exactly the same results » and the mathematical model of a natural phenomenon is considered as deterministic if the conditions of existence and uniqueness of solutions are satisfied, which is generally the case for models using systems of differential equations.

The physical determinism is very different. For many reasons, for instance because of the motions of planets, it is impossible to do twice **exactly** the same experiment. Thus a useful physical definition of determinism must be: « Two experiments with almost exactly the same initial and limit conditions must give almost exactly the same results ». In other words the **stability** of a phenomenon is an essential condition of the usefulness of the idea of determinism. For unstable phenomena, as soon as we consider durations longer than the time of divergence, a statistical analysis if more useful and more efficient than a deterministic analysis.

Let us consider the example of the kinetic theory of gas that have been studied at length by Poincaré [3]. The equations of motion of the kinetic theory of gas are as mathematically deterministic as those of Celestial Mechanics, however the instability is so large that it is impossible to predict the motion of a molecule after a few collisions, the slightest initial error would lead to major differences. This ''sensitivity to initial conditions'' is what is called today ''deterministic chaos '', it leads to statistical parameters as the temperature, the density and the pressure whose system of partial differential equations is based on a statistical analysis of the kinetic theory of gas. Of course that statistical system cannot be with an infinite accuracy, but the Avogadro number is so large that the statistics can be very accurate and the aerodynamicists use their equations as if they were absolutely accurate and deterministic.

We must notice that it is the instability of the phenomenon and its ''deterministic chaos'' that give a meaning to the statistical elements called temperature, pressure etc. Without them, it would be meaningless to apply the methods of averaging of the theory of probability to the analysis of the phenomenon of interest.

But Poincaré has gone further and has analysed the theory of quanta [4]. He has recognized that the discontinuity of quanta was a necessity :

« Donc, quelle que soit la loi du rayonnement, si l'on suppose que le rayonnement total est fini, on sera conduit à une fonction w présentant des discontinuités analogues à celles que donne l'hypothèse des quanta »[5].

« Thus, for any law of radiation, if we assume that the total radiation is finite, we will be led to a fonction w with discontinuities similar to these given by the hypothesis of quanta »[5].

In agreement with his analysis of kinetic theory of gas, Poincaré emphasized the importance of statistics and probability in quantum theory, but he has not gone until the point later reached by Heisenberg : the undeterminism as a principle. In 1910 that question was not sufficiently studied and understood.

4. Determinism, chaotic motions and arrow of time

Determinism, chaotic motions and arrow of time: these three questions have many surprising connections and we will again find here Henri Poincaré many times.

Many phenomena of day-to-day life are, or at least seems, irreversible: we remember the past and ignore the future, we are growing old, the heat always goes from hot bodies to colder ones and the sugar disappears into the coffee... However, the known physical laws are reversible.

This paradox has led to many researches and many controversies at the limit of philosophy, the classical answers are incompatible with each other and neglect or underestimate the importance of chaos that is certainly the essential reason of existence of physical irreversibility.

The second principle of thermodynamics is the main basis of physical irreversibility, but it is generally considered as a ‘’principle’’and not as a ‘’law’’. This linguistic subtlety allow us to write that all known scientific laws can always be expressed in a reversible form, a form in which past and future are symmetrical.

This leads to well-known symmetrical properties. For instance if , in the Solar System, we reverse the velocities of all planets and satellites, the orbits will remain the same and will be described in the other direction.

This beautiful symmetrical property is contradicted by so many ordinary irreversible phenomena that to some people irreversibility seems to be an inner property of matter and nature, but we will see that such a supposed property is not necessary.

Besides the second principle of thermodynamics, the corresponding increase of entropy and the related biological phenomena, the physicists consider that the major irreversible phenomena are:

A) The expansion of Universe.

B) The black holes: light and matter fall into black holes, they never escape from them.

C) The propagation of light: by diverging waves and never by converging waves.

There are some remarkable connections between these irreversible phenomena.

Let us consider for instance a lake in a cold country, it freezes each autumn and thaws each spring.

Ice is much more organized than water, it is then during freezing that the entropy of the lake decreases.

What happens then? The lake cannot be considered as an isolated system and, during the long and starry nights of November and December, it sends towards space a huge quantity of infra-red photons that carry away its heat and its entropy.

What happens to these photons? Because of the expansion of Universe, most of them will never arrive anywhere, they will wander forever in an always emptier space...

If Universe was static, the number of arriving photon would equal this of departing photons and the lake could not freeze.

There is however a question: if Universe is dense enough, it will not expand forever and in some tens of billions of years it will begin to contract. What will then happen? Some theoreticians think that then the time will reverse, we will escape from grave and return to childhood! This is really surprising and it is more probable that the black holes, so rare today, will then be very numerous. They will take the place of expansion and most wandering photons will fall into a black hole and will disappear forever. The lakes will continue to freeze each autumn...

4.1. A concrete example

Let us open the communication between two neighbouring vessels full of gas. The Brownian motion will egalize the temperatures, the pressures and the compositions while the opposite evolution never appears.

However :

A) The Brownian motion and the kinetic theory of gas are conservative and reversible, as conservative and reversible as Celestial Mechanics itself.

B) Henri Poincaré has demonstrated that for bounded and conservative systems, almost all initial conditions lead to an infinite number of returns in the vicinity of initial conditions [6] (the mathematicians specify: ``in any vicinity of initial conditions'').

These returns to the vicinity of initial conditions are of course contradictory with the equalization of temperatures, pressures and compositions.

4.2. Classical and unsatisfactory answers

A) ''There exist perhaps some very small, irreversible and dissipative hidden phenomena that forbid the application of Poincaré results...''

This rejection of a major symmetry of nature is not justified and we will see that our present knowledge is sufficient for the resolution of the observed contradiction.

B) ''For a given phenomenon the notion of trajectory remains accurate for only its time of divergence that is about fifty or one hundred « Liapunov times » and much less than the Poincaré return time that has never been observed in this type of experiment...''

This answer is true but insufficient. The impossibility of accurate long-term computations of future evolution doesn't resolve the contradiction.

C) ''In principle Poincaré is right and for strictly isolated systems there is indeed this mysterious correlation between initial and final conditions (after the

Poincaré return time). But our systems are not strictly isolated and even very small perturbations, such as the attraction of planets, destroy this correlation...''

These « mysterious correlations » are imaginary and it is in a natural fashion that the system returns towards all states attainable from the given initial conditions. The « very small perturbations » will not modify the order of magnitude of the Poincaré return time, even if it is true that they can modify very much the evolution in a relatively short interval of time (a few « Liapunov times ») and thus contribute to the disparition of correlations.

4. 3. The likely answer

It is because a system is « sensible to initial conditions » and because it depends of billions of parameters, while we measure only a few of them, that we ascertain an appearance of irreversibility and that the Poincaré return time is very large, much larger than the age of Universe.

We thus reach the physical irreversibility of our experiments in spite of reversible and conservative laws.

Notice that for non-chaotic evolutions, for instance for periodic or quasi-periodic evolutions, the deterministic predictions can be excellent, even if the knowledge of initial conditions is weak. A solution of these types has a natural reversibility and remains in a very small part of phase space, a part much smaller than that corresponding to chaotic motions.

The chaotic evolutions compensate their impossibility of long-term deterministic predictions by excellent long-term statistics predictions (notice the similarity with quantum mechanics). This excellency is related to the chaos itself that introduces randomness permanently and, even if it is impossible to predict the future motion of a given molecule in the Brownian motion, we can modelize very accurately the statistical elements such as the temperature and the pressure.

The following extremly simplified model, with only didactic purpose, will help to understand these questions.

Notice that this model satisfies practically the Boltzmann hypothesis of « molecular chaos » (no correlation between successive variations) but reaches opposite conclusions : the molecular chaos doesn't forbid the Poincaré return.

4.4. A simplified model

4.4.1. Numerical notations

In this example with many very large and very small numbers we will use the notation ''by figures and sizes'' with the letter p for ''positive power of ten'' and the letter n for ''negative power of ten''.

Hence, for instance :

$$6.02 \text{ p23} = \text{Avogadro number} = 6.02 \times 10^{23}$$

$$1.66 \text{ n24} = \text{reciprocal of the Avogadro number} = 1.66 \times 10^{-24}$$

Notice that the ‘’figures’’, i.e. here 6.02 and 1.66, are **always between one and ten.** This gives an unambiguous definition of the ‘’sizes’’(here p23 and n24). The size is the main element of very large and very small quantities, it is even very often their only known element.

4.4.2. The simplified model

Let us consider one billion billion molecules (that is p18 molecules with the above notation by figures and sizes). This number is the number of molecules in $37mm^3$ of air in ‘’normal conditions’’(0° and 1013 millibars), which is a very small volume. In most experiments the effects will be even larger.

These p18 molecules will be put into the two identical vessels A and B and will be numbered from 1 to p18.

The evolution will be : at each step an integer number between 1 and p18 will be chosen and the corresponding molecule will be transferred from its present vessel to the other one.

The rate of these exchanges can be, for instance, one million billion (that is p15) per second.

We will consider the temperature as constant and we will measure only the number of molecules in the two vessels A and B, that is the local pressure. We can for instance start with the following initial pressures :

$$P_A(0) = 1.4 \text{ bar} \ ; \ P_B(0) = 0.6 \text{ bar} . \qquad (1)$$

Hence proportionally the initial share will be 7 p17 molecules for the vessel A and 3 p17 molecules for the vessel B.

What will be the evolution?

4. 5. Evolution of the pressures P_A and P_B

The total number of molecules is constant and thus at any time t :

$$P_A(t) + P_B(t) = P_A(0) + P_B(0) = 2 \text{ bars} . \qquad (2)$$

It is then sufficient to consider only the evolution of the pressure $P_A(t)$.

An essential question is the mode of choice of the successive 18 digit integer numbers.

We can consider a purely random choice (it is the ‘’molecular chaos’’ of Boltzmann), but, because of usual philosophical objections, we will also consider the following deterministic choice : the k^{th} choice wil be given by the decimals of rank $(18k - 17)$ to $18k$ of a given real number x.

For instance, with $x = \pi$, that is :

$$x = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643... \qquad (3)$$

the first choice will be : 141 592 653 589 793 238.

The purely random choice leads to a simple analysis. With P_A and its variation δP_A expressed in bars we obtain the following : at each step we have $\delta P_A = \pm 2$ n18 with the probability $(P_A / 2)$ for $\delta P_A = - 2$ n18 and the remaining probability $1 - (P_A / 2)$ for $\delta P_A = + 2$ n18.

Hence the average evolution $P_{AM}(t)$ is given at each step by :

$$\delta P_{AM} = (1 - P_{AM}) \times 2\ n18 \qquad (4)$$

that is, after k steps :

$$P_{AM.k} = 1 + [\ P_A(0) - 1].[1 - 2\ n18]^k \qquad (5)$$

which, with $P_A(0) = 1.4$ bar and with p15 steps per second, gives :

$$P_{AM}(t) = [1 + 0.4 \exp\{- 0.002\ t\}] \text{ bar} ; \quad \text{with t expressed in seconds} \qquad (6)$$

P (one minute) = 1.3548 bar
P (one hour) = 1.000 298 6 bar (7)
P (one day) = (1 + 4 n76) bar ; practically one bar .

Thus the average pressure $P_{AM}(t)$ decreases and converges exponentially towards one bar, it gives an impression of irreversibility. However we must also take account of the variance V(t) of the pressure.

That second study is of course more complex, the $(k + 1)^{th}$ step gives :

$$V_{k+1} = (1 - 4\ n18)V_k + 4\ n36\ [1 - (P_{AM.k} - 1)^2] . \qquad (8)$$

Hence, with (5) and with $V_0 = 0$:

$$V_k = [n18 - 0.16\ (1 - 2\ n18)^{2k} + (0.16 - n18)\ (1 - 4\ n18)^k]\ \text{bar}^2 . \qquad (9)$$

With the time t expressed in seconds, the exact expression (9) gives almost exactly :

$$V(t) = [1 - (1 + 0.000\ 64\ t).\exp\{- 0.004\ t\}]\ n18\ \text{bar}^2 \qquad (10)$$

The variance V(t) has a monotonic evolution, it increases from zero to n18bar^2 and at t = 1000 seconds, it has already 97% of its final value.

The main result is that the variance V(t) will remain forever very small.

The standard deviation $\sigma(t)$, the square root of the variance, will also remain forever very small , its maximum is n9 bar that is one billionth of a bar or one deci-millipascal (one pascal = 1 Pa = 1 N/m^2 = n5 bar).

If we measure the pressure with an excellent accuracy of the order of one millipascal (i.e. ten standard deviations), we will notice from time to time a fluctuation with respect to the average solution. For instance fluctuations larger than 7.94 standard deviations have an average frequency of one per two years and an average duration of 0.127 microsecond only...

If we measure the pressure with the very good accuracy of five millipascals, i.e. fifty standard deviations, we will never notice any fluctuation and the evolution will appear as irreversible. Indeed, there is only the probability n200 (that is 10^{-200}) that the first fluctuation of five millipascals appears before the time t = 4.625 p329 seconds, that is 1.465 p322 years.. (the age of Universe is only about 1.5 p10 years...).

In these conditions the return time of Poincaré is extremly large and purely theoretical, but it can be computed : if again we neglect a probability of n200 (which correspond to the ‘’threshold of certainty of observable Universe’’); the first Poincaré return to the pressure 1.4 bar will occur after 10^R seconds with :

$$35\ 735\ 000\ 089\ 859\ 502 \le R \le 35\ 735\ 000\ 089\ 859\ 706 . \qquad (11)$$

Let us recall that all these results correspond to the random choice of successive exchanges and we have also to consider the case of deterministic choices, as explicited with equation (3). The computations (4)-(11) gives then that the values of x that satisfy $0 \le x \le 1$ and that doesn't satisfy (11) have a total measure smaller than n200. This set of values is then completely negligible even if many remarkable values of x, such as 1/3 = 0.333 333..., belong to that set.

4. 6. A final remark

The equations (6) and (7) of evolution of the average pressure $P_{AM}(t)$ show clearly an irreversible phenomena but can also give the false impression of an essential dissymetry between the past (far from equilibrium) and the futur (close to equilibrium).

On the contrary there is a strict past-future symmetry and if we only know, in this experiment, that at the time t_1 and the ulterior time t_2 the pressures $P_A(t)$ are P_{A1} and P_{A2} the evolution of the average pressure $P_{AM}(t)$ for $t_1 \le t \le t_2$ is given by :

$$P_{AM}(t) = [1 + P_1 \exp\{0.002(t_1 - t)\} + P_2 \exp\{0.002(t - t_2)\}] \text{ bar}$$

with t_1,t,t_2 expressed in seconds and P_1,P_2 given by $P_{AM}(t_1) = P_{A1}$; $P_{AM}(t_2) = P_A$ (12)

Thus even if the evolution (6), (7) is irreversible the past-future symmetry is conserved. Look, for instance, for the arrival at a Poincaré return at t_2 with t_1 negative and extremly far.

5. Conclusions

All along the nineteenth century determinism was the dominant idea of Science, but its absolute reign was questionned by Henri Poincaré for both philosophical and scientific reasons.

The twentieth century has seen the continuous development of the new Poincaré's ideas. The chaotic motions, the strange attractors, the sensitivity to initial conditions appear now in almost all domains of science and technology and have completely modified our understanding of nature. The corresponding time of divergence is a rapidly increasing function of the scale of the phenomenon of interest ; extremly short at atomic scale – in agrement with the statistic and probabilistic character of quantum mechanics – it is usually a few seconds or a few minutes for ordinary turbulent flows, about fifteen days for meteorology and several millions of years for the astronomical motions of our Solar System.

The chaotic motions destabilize the individual elements (position and velocity of a particle) but stabilize the corresponding mean statistical elements (pressure, temperature) that become the basic elements of the larger scale. Phenomena are thus nested in one another until the astronomical scale that uses the notion of ''center of mass of a celestial body'' and studies its motions without being disturbed by the inner motions and the streams of this body.

The chaotic motions are strongly connected with the arrow of time and the irreversibility of the second principle of thermodynamics. They all agree with **our**

measures and **our experiments** that are, by far, neither long nor numerous enough to lead to a contradiction. However some very small temporary fluctuations appear from time to time in very accurate experiments.

Thus, the paradox of reversible laws associated with irreversible phenomena can be explained without ‘’small hidden irreversibility’’, ‘’perfect isolation’’ and/or ‘’hidden correlations’’. The main reasons of physical irreversibility are the chaotic character and the very large number of parameters of irreversible systems.

The Boltzmann hypothesis of «molecular chaos» is excellent and allows very accurate computations. The correlations will not increase slowly and insidiously after a very long time and we can almost write that the return of Poincaré occurs by chance which requires such a large delay, much larger than the age of Universe, that the corresponding decrease of entropy never appears in our experiments.

If we meet so many phenomena with an increase of entropy, it is because desequilibriums are easy in our world: the smallest valley has a sunny side and a shady one... The fundamental reason is our existence in the middle of a giant stream of energy that arrives continuously from the burning Sun and escapes to the frozen space.

The Poincaré return time is exponentially connected with the number of independant parameters of the system of interest and we can thus write :

‘’If, after the usual mathematical simplifications (integrals of motion, symmetries, decomposability, etc.) a system :

A) remains with a large number N of independant parameters.

B) is sensible to initial conditions.

C) is analysed through statistical parameters as the temperature and the pressure.

Then its evolution will physically appear as irreversible for measures of relative accuracy worse than ($50 / \sqrt{N}$) even if its laws are mathematically reversible and conservative.

These successive revisions of the ideas of determinism and irreversibility open the way for a reconciliation of Science with the subjective notions of freedom,will, free will, the essential pillars of human dignity. It seems impossible to demonstrate scientifically either the exitence or the inexistence of freedom, that is something very different from random, and the present tendency is to assume a new postulate, as unprovable as the axioms of geometry or arithmetic: «There is a source of freedom in each human being ».

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