On the Bag Models Based on the Singular Solution of Yang–Mills Equations

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My report is devoted to the problem of constructing of the model of quark bag on the basis of singular solutions of classical Yang-Mills equations. These solutions had been obtained in works of F.A.Lunev [1] in gauge-invariant approach to Yang-Mills theory by using the analogy between these solutions and black-hole solutions in general relativity. The idea to use these solutions as the basis of the confinement of quarks is proposed in [2]. But on the way of realization of this idea the very essential problem arises. The self-energy of the gluon bag that confines the quark in the closed domain is infinite. This problem seems to be very typical for the bag model, but in our model this problem can be solved.

The basic assumption is that quarks in zero approximation move in a certain effective YM potential that is a solution to classical YM equations with singularity on the sphere. Our model obviously can be derived from QCD by quantization in neighborhood of such singular solution as zero approximation. Further correction can be also obtained in a systematic way. Indeed let us consider "partition function" that is represented as path integral

$$Z = \int DA\, D\bar{\Psi} D\Psi exp\{i \int_0^T dt \int d^3x (L_{YM}(A) + L_{ferm}(\bar{\Psi}, \Psi, A))\}. \tag{1}$$

We assume that the main contribution in this path integral is given by trajectories close to classical solution with singularity on the sphere. Applying stationary phase method one gets the follow expression in zero approximation

$$Z = \sum_{1 \le k_s \le n_s} \int_{\substack{zero \\ modes}} e^{-i(E_{YM}(\mathcal{R}) + \sum_q \sum_s k_s E_s(\mathcal{R}, m_q))T}, \tag{2}$$

where

- $E_{YM}(\mathcal{R})$ is self-energy of the classical YM fields configuration that has singularity on the sphere ;
- $E_s(\mathcal{R})$ are positive eigenvalues of Hamiltonian of color Dirak particle

$$i\gamma^0 \vec{\gamma} \vec{\nabla}(A_{cl}) - \gamma^0 m_q;$$

- n_s is doubled multiplicity of the eigenvalue E_s ;
- $\mathcal{R}$ is a radius of singularity.

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Applying again stationary phase method, one gets the formula to determine the mass of “hadron”

$$M^{theor} = E_{YM}(\mathcal{R}_0) + \sum_{quarks} E(\mathcal{R}_0, m_q)$$

where $\mathcal{R}_0$ is defined from an equation

$$\frac{\partial}{\partial \mathcal{R}}[E_{YM}(\mathcal{R}) + \sum_{quarks} E(\mathcal{R}_0, m_q)]\Bigg|_{\mathcal{R}=\mathcal{R}_0} = 0.$$

In this formula the quarks contribution $E(\mathcal{R}_0, m_q)$ is defined by a standard numerical procedure. Details of this calculation you can find in our work [3].

But now I want to discuss the contribution to the mass formula from YM fields. This is $E_{YM}(\mathcal{R})$. The problem is that quantity $E_{YM}(\mathcal{R})$ is divergent due to singularity at $r = \mathcal{R}$ and must be regularized.

But before this I wont shortly discuss the theory of singular solution of YM equation.

For simplicity, let us consider the three dimensional $SU(2)$ YM theory. There are two approaches to this theory: the first one is a traditional approach in terms of potential A_μ of YM fields and the second one is a gravity-like approach proposed by Lunev [1]. This approach operates with a second rank tensor

$$G_{ij} = -\frac{1}{2}\varepsilon_{ikl}\varepsilon_{jmn}\, tr\, F^{kl}F^{mn}, \tag{1}$$

where F_{ij} is field strength tensor. G_{ij} is treated as metric of some manifold $\mathcal{M}$. Description of the “dynamics” of this manifold is obsolutely equivalent to description of YM fields in terms of potential. To make the discussion more clear both of these approaches are used in parallel way. It appears that the lagrangian and equation of motion can be written in both approaches as in Table 1.

Substituting in these equations the spherically symmetrical ansatz one gets absolutely identical equations. It is not surprising because these approaches are absolutely equivalent.

These equations have singular on the sphere solutions which have quasi-Schwarzschild behavior near the singularity

$$H(r) \sim \frac{\sqrt{2}}{1 - r/\mathcal{R}}\,, \; r \to \mathcal{R} - 0.$$

The functional of classical energy of YM field

$$E_{YM}[H(r)] = \frac{4\pi}{g^2}\int^R \{\frac{1}{2}(H')^2 + \frac{(H^2-1)^2}{4r^2}\} \to \infty$$

is divergent on this solution due to singularity of A_{cl}, and the procedure of regularization must be suggested.

We suggest a very simple method to solve this problem. From methodological point of view this method is a modification of the well-known method of regularization by high covariant derivative [4].

Table 1: R_{ij} is a Ricci tensor of manifold $\mathcal{M}$ corresponds with metric G_{ij}. R is a scalar curvature. There g^{ij} is a metric of the Euclidean space.

	Approach I	**Approach II**
Lagrangian of the system	$L = -\frac{1}{16g^2} tr(F_{ij}F_{ij})$	$L = -\frac{1}{16g^2}(\sqrt{G}R + \lambda g^{ij}G_{ij})$
Equation of motion	$D_iF_{ij} = 0$	$R^{ij} - \frac{1}{2}RG^{ij} = \lambda\frac{\sqrt{g}}{\sqrt{G}}g^{ij}$
Spherically symmetrical ansatz	$A_i^a = \varepsilon_{aij}\frac{x^j}{r^2}(1 - H(r))$	$G_{rr} = A(r),\ G_{\theta\theta} = B(r),$ $G_{\phi\phi} = B(r)\sin^2\theta$
Resulting equation	$r^2H'' = H(H^2 - 1)$	After changing of variables $K = \sqrt{B} \quad H = K'/\sqrt{A}$ one gets $r^2H'' = H(H^2 - 1)$
Regularization part in lagrangian	$\Delta L = -\frac{i\varepsilon^2}{12g^2} tr(F_{ij}F_{jk}F_{ki})$	$\Delta L = -\frac{1}{16g^2}(-2\Lambda)\sqrt{G}, \quad \Lambda = \frac{1}{\varepsilon^2}$

Instead of initial theory with lagrangian L, we would describe a slightly-modified theory with lagrangian $L^\varepsilon = L + \varepsilon\Delta L$. The modified theory has to satisfy this obvious requirements:

1. If ε tends to the zero, L^ε tends to L : $\quad L^\varepsilon \quad \xrightarrow{\varepsilon\to 0} \quad L.$
2. L^ε has a maximum symmetries of L.
3. Theory with L^ε set has a set of solutions $H_\varepsilon(r)$ that tend to the H(r) almost everywhere if ε tends to the zero.
4. Functional of classical energy of modified theory on the solutions $H_\varepsilon(r)$ has to be finite for any nonzero ε.
5. The correction ΔL must have the same order by inverse g that has initial lagrangian L.

It tern out that it is possible to find a correction that satisfies these requirements. You can see this in the last section of table 1. This correction isn't surprising, in gravity-like approach this correction is the well-known Λ-part. This one has a $\frac{1}{g^2}$ order since this term can be expressed the following form

$$tr(D_iF_{jk} \times D_iF_{jk}) - tr(D_iF_{ik} \times D_JF_{jk}) = -4i\, tr(F_{ij}F_{jk}F_{ki}).$$

Let us variate the new modified lagrangian L^ε and substitute the spherically symmetrical ansatz in the results one gets the following expression

$$(1 - \frac{\varepsilon^2}{r^2}(H_\varepsilon^2 - 1))r^2H_\varepsilon'' = H_\varepsilon(H_\varepsilon^2 - 1) + \frac{\varepsilon^2}{r^2}(rH_\varepsilon')^2H_\varepsilon - 2\frac{\varepsilon^2}{r^2}rH_\varepsilon'(H_\varepsilon^2 - 1).$$

If ε tends to zero this equation becomes a Wu-Yang equation that has singular on the sphere solutions. But this equation has solutions suggested to contributions 3) and 4) but nonsingular on the sphere. In point $r = R$ these solutions have asymptotic

$$H_\varepsilon(r) \sim \sqrt{1 + R^2/\varepsilon^2} + C(1 - r/R)^{2/3} + \dots$$

where C is a constant.

If we substitute these solutions in new functional of classical energy

$$E^\varepsilon_{YM}[H(r)] = \frac{4\pi}{g^2} \int^R \{ \frac{1}{2}(1 - \frac{\varepsilon^2}{r^2}(H^2 - 1))(H')^2 + \frac{(H^2-1)^2}{4r^2} \}$$

we get a finite result. But if ε tends to zero, result tends to infinity. This is a principal point of our contruction. On the set of solutions $\{H_\varepsilon(r)\}$ the functional of classical energy is a function of ε. Getting the asymptotic at $\varepsilon \to 0$ of this function is very nontrivial operation but result is following

$$E^\varepsilon_{YM}[H_\varepsilon(r)] \xrightarrow{\varepsilon \to 0} \frac{1}{g^2}(constant_1 + O(\varepsilon)).$$

Now if we suggest that g is

$$g = \frac{constant_2}{\varepsilon^2} + O(\frac{1}{\varepsilon})$$

one gets the finite expression for the E_{YM}

$$E^\varepsilon_{YM}[H_\varepsilon(r)] = B_{phys} R^3 + O(\varepsilon).$$

Now as we expect in the previous works [2, 3] the regularized self-energy of the bag gives a polynomial contribution in mass formula and is proportional to the volume of bag. In order to achieve the best agreement with the experimental data we choose the following volum for the constant B. Our result is in agreement with experiment data with accuracy 3-7 per cent for all hadron mass except those of light pseudoscalar mesons. This accuracy is a maximal possible accuracy for any constituent quark model in which interaction between the quark isn't taken into account.

References

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