Nonstationary Selfconsistent Solutions in Quantum Field Theory

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Polaron problem is considered. Bogoliubov group variables for the space-time translations group are defined. The scheme of secondary quantization for the scalar field interacting with charged quantum field is proposed. Scalar field is assumes to have nonzero classical component. Polaron is treated as a result of interaction of charged particle and neutral field classical component. The system state space is constructed. Expressions for integrals of motion in zero-point order with respect to inverted powers of coupling constant are given as a derivatives with respect to symmetry group generators.

1. INTRODUCTION

The problem of accurate account of conservation lows in quantum theory when methods of approximate calculations are used has been solved by N.N.Bogoliubov. He considered the motion of classical particle in quantum field. The system was invariant with respect to space-time translations, so the momentum of the system was integral of motion. Bogoliubov proposed to take into consideration some new variables which had sense of invariance group generators. New variables turn out to be cyclic. And derivatives with respect to new variables coincided with system momentum, so operators of Hamiltonian and momentum commuted and momentum conservation low was accomplished at once.

Lately it was shown that such scheme is applicable for any Lee group. However it is difficult to describe the systems including time translations, course Hamiltonian structure as generator of time translations becomes clear only after solution of equation of motion, so the task was a rather indefinite. Hence we need to define group variables together with developing of perturbation theory, and to specify formulae of variables substitutions step by step, in according with forthcoming to accurate solutions of field equations. In [8] such scheme of Bogoliubov transformations was realized for the Poincare-invariant self-acting scalar field.

In present work we come back to polaron problem and treat it as a problem of quantum field theory. We enter Bogoliubov group variables for the system of interacting neutral and quantum charged scalar fields in space-time of (1+1)-dimension and consider scalar field quantization in the neighborhood of nonzero classical component. The system is assumed to be invariant with respect to translations group transformations.

2. BOGOLIUBOV GROUP VARIABLES

Let's consider $(1+1)D$ system of charged and neutral scalar fields. Operators $\hat{\Psi}, \hat{\Psi}^*$ correspond to charged field and ϕ is neutral field.

Field interactions leads to appearance of classical component of neutral field. Bogoliubov transformation permits to separate this component:

$$\xi(x) = gv(x^{'}) + u(x^{'}), \tag{1}$$

where $g \gg 1$,

$$x^{'\alpha} = x^\alpha - \tau^\alpha, \quad \alpha = 1, 2,$$

τ^α-new variables connected with space-time translations. τ^α are considered as independent, so the right part of eq(1) contains on 2 variables more than the left part, and it is necessary to equalize the number of variables. To that end to additional conditions are imposed on $u(x^{'})$ function:

$$w(N^a, u) = \int (N^a_n(x^{'})u(x^{'}) - N^a(x^{'})u_n(x^{'}))d\lambda = 0. \tag{2}$$

Symbol f_n denote the normal derivative of function f alone the integration line C which is defined as a straight line with a normal $n = (n^\alpha), n^2 = 1$ for simplicity. Equation of the line C

$$x^\alpha = e^\alpha \lambda$$

is determined by vector n $e = (e^\alpha), e^2 = -1$ orthogonal to n. Using substitution (1) for $\xi(x)$ in (2) one can obtain functional dependence τ^α with respect to ξ, ξ_n in differential form:

$$\frac{\delta \tau^c}{\delta \xi(x)} = -\frac{1}{g} Q^c_b N^b_n(x^{'}), \quad \frac{\delta \tau^c}{\delta \xi_n(x)} = \frac{1}{g} Q^c_b N^b(x^{'}),$$

where $Q^a_b = \delta^a_b - \frac{1}{g} R^a_c Q^c_b, R^c_\alpha = -w(N^c_\alpha, u)$. From now onwards $w(N^a, v_\beta) = \delta^a_\beta$, it is possible to satisfy by appropriate N^a choice. Varying $\xi(x)$ and $\xi_n(x)$ we have:

$$\frac{\delta}{\delta \xi(x)} = \frac{\delta}{\delta u(x')} + \frac{\delta \tau^\alpha}{\delta \xi(x)} \Big(S_\alpha + \frac{\partial}{\partial \tau^\alpha}\Big), \qquad \frac{\delta}{\delta \xi_n(x)} = \frac{\delta}{\delta u_n(x')} + \frac{\delta \tau^\alpha}{\delta \xi_n(x)} \Big(S_\alpha + \frac{\partial}{\partial \tau^\alpha}\Big),$$

where

$$S_\alpha = \int \Big(u_\alpha(x') \frac{\delta}{\delta u(x')} + u_{\alpha n}(x') \frac{\delta}{\delta u_n(x')}\Big) d\lambda.$$

Here the following equation is used:

$$\int \Big(v_\alpha(x') \frac{\delta}{\delta u(x')} + v_{\alpha n}(x') \frac{\delta}{\delta u_n(x')}\Big) d\lambda = 0. \tag{3}$$

This equation is the immediate consequence of additional conditions (2) and relationship $w(N^a, v_\beta) = \delta^a_\beta$.

Let's define operators:

$$\hat{q}(x) = \frac{1}{\sqrt{2}} \Big(\xi(x) + i \frac{\delta}{\delta \xi_n(x)}\Big), \qquad \hat{p}(x) = \frac{1}{\sqrt{2}} \Big(\xi_n(x) - i \frac{\delta}{\delta \xi(x)}\Big). \tag{4}$$

They are selfconjugated and satisfy commutation relations

$$[\hat{q}(x^{'}), \hat{p}(x^{''})] = i\delta(x^{'} - x^{''}),$$

so in is possible it treat them as coordinate and momentum operators of field oscillators.

However there is another pair of selfconjugated operators which is satisfy the same commutation relations. So the number of possible field states turns out to be doubled. Field state number reduction is possible to carry out using holomorphic representation for example, see [8].

In the terms of new variables operators $\hat{q}(x)$ and $\hat{p}(x^{)}$ looks like:

$$\hat{q}(x) = gF(x') + Q(x') + \frac{1}{g}A(x'), \qquad \hat{p}(x) = gF_n(x') + P(x') + \frac{1}{g}A_n(x'),$$

where

$$F(x') = \frac{1}{\sqrt{2}}(v(x') + N^b(x')\Phi_b), \qquad A(x') = \frac{1}{\sqrt{2}}N^b(x')Q_b^c T_c,$$

$$Q(x') = \frac{1}{\sqrt{2}}\Big(u(x') + i\frac{\delta}{\delta u_n(x')} - N^b(x')r_b\Big),$$

$$P(x') = \frac{1}{\sqrt{2}}\Big(u_n(x') - i\frac{\delta}{\delta u(x')} - N^b_n(x')r_b\Big),$$

$$T_\gamma = K_\gamma + R^a_\gamma r_a, \qquad r_\alpha = R^c_\alpha \Phi_c, \qquad K_\alpha = iS_\alpha + i\frac{\partial}{\partial \tau^\alpha}.$$

$\frac{\partial}{\partial \tau^\alpha}$ enter in integrals of motion in the order $\frac{1}{g}$ only. In order to take into account all interaction effects earlier the canonical transformation is made: $i\frac{\partial}{\partial \tau^\alpha} \to g^2\Phi_\alpha + i\frac{\partial}{\partial \tau^\alpha}$, and then K_α becomes to be $K_\alpha + g^2\Phi_\alpha$.

3. SCHEME OF PERTURBATION THEORY

The dynamic of the system is determined by Lagrangian (function arguments are omitted):

$$L = \frac{1}{2}\int(\phi_n^2 - \phi_\lambda^2 - \mu^2\phi^2)d\lambda +$$

$$+g^2\int(\Psi_n^*\Psi_n - \Psi_\lambda^*\Psi_\lambda - m^2\Psi^*\Psi)d\lambda - -g\int \Psi^*\Psi\phi d\lambda. \tag{5}$$

Action is invariant with respect to space-time translations, so energy and momentum are to integrals of motion. Now it is possible to quantize via substitution

$$\phi \longrightarrow \hat{q}, \qquad \phi_n \longrightarrow \hat{p}, \qquad \phi_\lambda \longrightarrow \hat{q}_\lambda,$$

and we have the following expressions of the integrals of motion:

$$E = H = \frac{1}{2}\int(\hat{p}^2 + \hat{q}_\lambda^2 + \mu^2\hat{q}^2)d\lambda +$$

$$+g^2\int(\hat\Psi^*_n\hat\Psi_n+\hat\Psi^*_\lambda\hat\Psi_\lambda+m^2\hat\Psi^*\hat\Psi)d\lambda+g\int\hat\Psi^*\hat\Psi\hat q d\lambda,$$

$$P=\int\hat p\hat q_\lambda d\lambda+g^2\int\hat\Psi^*_n\hat\Psi_\lambda d\lambda+g^2\int\hat\Psi_n\hat\Psi^*_\lambda d\lambda.$$

Using expressions for $\hat p$, $\hat q$, one can expand integrals of motion into the series with respect to inverted powers on coupling constant g:

$$O=g^2O_{-2}+gO_{-1}+O_0+\cdots.$$

Let's take notice that H_{-2},P_{-2} don't depend from neutral field operators.

In prospect we will consider only one-particle conditions of charged field

$$|1>=\int\hat\Psi^*(x)f(x)d\lambda|0>,\qquad <1|=<0|\int\hat\Psi(x)f^*(x)d\lambda$$

and

$$[\hat\Psi(x),\hat\Psi^*(x^{'})]=-iD(x-x^{'}).$$

If state vectors is $\Psi=\Psi_{ch}\otimes\Psi_n$, in the first approximation, then averages of $<H_{-1}>_{ch}$, $<P_{-1}>_{ch}$ are reduced to linear forms with respect to neutral field operators. There is no normalizible eigenvectors of operators O_{-1}, so perturbation theory is applicable only is this operators are equal zero.

If the following boundary condition is performed $F_\lambda Q\,|_{\partial c}=0$, than

$$H_{-1}=\frac{1}{\sqrt2}\int\Big(F_n(u_n-i\frac{\delta}{\delta u}-r_bN^b_n)+B(u+i\frac{\delta}{\delta u_n}-r_bN^b)\Big)d\lambda,$$

where $B=-F_{\lambda\lambda}+\mu^2F+<\hat\Psi^*\hat\Psi>=-F_{\lambda\lambda}(x^{'})+\mu^2F(x^{'})+V(x^{'})$.

Let's suppose the following equality to be true:

$$F_n=c^\alpha v_\alpha,\quad B=-c^\alpha v_{\alpha n},\tag{6}$$

than the first order of Hamiltonian tuns out to be (3), and condition of perturbation theory application is accomplished. As a consequence of (6) we obtain equation for the classical component of neutral scalar field:

$$F_{nn}-F_{\lambda\lambda}+\mu^2F+V=0.$$

Further $F(x^{'})$ is considered as a solution of the equation

$$F_{tt}(x^{'})-F_{xx}(x^{'})+\mu^2F(x^{'})+V(x')=0\tag{7}$$

with given boundary conditions on *(Caushy problem)*, here

$$V(x)=-\int\int f^*(x_1)D(x_1-x)D(x-x_2)f(x_2)d\lambda_1d\lambda_2.$$

It is easy to show that $H_{-1}=0$ if $v_\alpha=N^c_\alpha\Phi_c$ it means that $F=cN^c\Phi_c$. From $F=\frac{1}{\sqrt2}(v+N^c\Phi_c)=\sqrt2v$ immediately follows that $c=\sqrt2$.

Analogously one can show that $P_{-1}=0$, if the following boundary conditions are accomplished $F_\lambda Q\,|_{\partial c}=0$.

It is possible to obtain equation for the function $f(x)$ describing one-particle charged field state. $\hat{\Psi}(x)$ satisfy the following equation (due to variational principle):

$$\hat{\Psi}_{tt} - \hat{\Psi}_{xx} + m^2\hat{\Psi} + v\hat{\Psi} = 0.$$

Than it is possible to represent $\hat{\Psi}(x)$ as a following:

$$\hat{\Psi}\left(x\right) = \hat{\Psi}_0\left(x\right) + \int D_c\left(x-y\right) v\left(y\right) \hat{\Psi}\left(y\right) dy, \tag{8}$$

where $\hat{\Psi}_0\left(x\right)$ - is a solution of the homogeneous equation.

One-particle state is:

$$\mid 1\rangle = \int f\left(x\right) \hat{\Psi}^*\left(x\right) \mid 0\rangle dx.$$

Denote

$$\langle 0 \mid \hat{\Psi}\left(x\right) \mid 1\rangle = \int \langle 0 \mid \hat{\Psi}\left(x\right) f\left(y\right) \hat{\Psi}^*\left(y\right) \mid 0\rangle dy = \bar{f}\left(x\right),$$

$$f(x) = ([] + m^2)\bar{f}(x).$$

From (8) we have:

$$\bar{f}\left(x\right) = \langle 0 \mid \hat{\Psi}_0\left(x\right) \mid 1\rangle + \int D_c\left(x-y\right) v\left(y\right) \bar{f}\left(y\right) dy.$$

$\bar{f}(x)$ satisfies to the equation

$$\left([] + m^2\right) \bar{f}\left(x\right) + v\left(x\right) \bar{f}\left(x\right) = 0.$$

In nonrelativistic limits $\bar{f}(x)$ could be represented as

$$\bar{f}\left(x\right) = e^{-imt}\bar{\xi}\left(x,t\right),$$

and it $\bar{\xi}\left(x,t\right)$ changes slowly in process of time, it means $\bar{\xi}_{tt}\left(x,t\right) = 0$, and for $\bar{\xi}\left(x,t\right)$ we have:

$$i\frac{\partial\bar{\xi}\left(x,t\right)}{\partial t} + \frac{1}{2m}\nabla_r\bar{\xi}\left(x,t\right) - \frac{1}{2m}v(x)\bar{\xi}\left(x,t\right) = 0.$$

4. SYSTEM STATE SPACE CONSTRUCTION

Neutral field dynamic properties in zero-point order with respect to inverted powers of coupling constant g are determined by operators H_0 and P_0. It is possible to show that after system state space construction those operators reduces to the derivatives with respect to symmetry group generators.

If $AF_\lambda\mid_{\partial c} = 0$, where $A = \frac{1}{\sqrt{2}}N^bT_b$, then

$$H_0 = in^\alpha\frac{\partial}{\partial\tau^\alpha} + \frac{1}{2}\int(P^2 + Q_\lambda^2 + \mu^2Q^2)d\lambda +$$

$$+ i\int\left(u_n\frac{\delta}{\delta u} + + u_{nn}\frac{\delta}{\delta u_n}\right)d\lambda + \int(N_n^c u_n - N_{nn}^c u)d\lambda r_c. \tag{3}$$

If coordinate and momentum operators of field oscillators looks like (4), then the number of possible neutral scalar field states is doubled over against real situation. That is why states number reduction is necessary.

Primarily let's analyze the number of independent variables. Original number of independent variables was ∞. After defining of Bogoliubov group variables (they are considered to be independent) the number became $\infty + 2$. This number was doubled due to determination of creation-annihilation operators: $(\infty + 2) * 2 = 2 * \infty + 4$. Additional conditions reduced the number of independent variables on 2, that is at present time the number of possible field states is $2 * \infty + 2$. Let's separate from field variables $u(x^{'})$ two variables r_a which has no any physical sense and are connected with the method of perturbation scheme realization. Then the state number is $2 * \infty$, and field is described via $w(x^{'})$ variables which are determined as the following:

$$u = w + N^a r_a, \quad u_n = w_n + N^a_n r_a,$$

$$\frac{\delta}{\delta u} = \frac{\delta}{\delta w} + \frac{\delta r_a}{\delta u}\frac{\partial}{\partial r_a}, \quad \frac{\delta}{\delta u_n} = \frac{\delta}{\delta w_n} + \frac{\delta r_a}{\delta u_n}\frac{\partial}{\partial r_a}.$$

In this case operators Q and P are:

$$Q = \hat{Q} + q, \quad P = \hat{P} + p,$$

where

$$\hat{Q} = \frac{1}{\sqrt{2}}\Big(w + i\frac{\delta}{\delta w_n}\Big), \quad \hat{P} = \frac{1}{\sqrt{2}}\Big(w_n - i\frac{\delta}{\delta w}\Big),$$

$$q = \frac{i}{\sqrt{2}}\frac{\delta r_a}{\delta u_n}\frac{\partial}{\partial r_a}, \quad p = -\frac{i}{\sqrt{2}}\frac{\delta r_a}{\delta u}\frac{\partial}{\partial r_a}.$$

Using variables w and w_n one can reduce of state number by the following way: let's describe the field states by the functionals in which $\frac{\delta}{\delta w}$ и $\frac{\delta}{\delta w_n}$ reduces to the operators:

$$\frac{\delta}{\delta w} \to \frac{\delta}{\delta w} - iw_n, \quad \frac{\delta}{\delta w_n} \to -iw.$$

Then

$$\hat{Q} \to \sqrt{2}w, \quad \hat{P} \to -i\frac{1}{\sqrt{2}}\frac{\delta}{\delta w},$$

and if $\frac{\partial r_a}{\partial u_n} w_\lambda|_{\partial c} = 0$, after states number reduction Hamiltonian H_0 in zero-point order with respect to coupling constant depends on extra variables r_a and derivatives with respect to translation group generators only.

$$H_0 = in^\alpha\frac{\partial}{\partial \tau^\alpha} + \frac{1}{2}\int(p^2 + q^2_\lambda + \mu^2 q^2)d\lambda +$$

$$+i\int(N^a_n\frac{\delta r_b}{\delta u} + N^a_{nn}\frac{\delta r_b}{\delta u_n})d\lambda r_a\frac{\partial}{\partial r_b} + \int(N^c_n N^a_n - N^c_{nn}N^a)d\lambda r_a r_c. \tag{10}$$

Analogously if $AF_n\,|_{\partial c} = 0$, $\hat{Q}p\,|_{\partial c} = 0$, $w\frac{\delta r_a}{\delta u}\,|_{\partial c} = 0$, $w_n\frac{\delta r_a}{\delta u_n}\,|_{\partial c} = 0$ and $ww_n\,|_{\partial c} = 0$, after reduction momentum turns out to be function of nonphysical variables and translations parameters only :

$$P_0 = ie^{\alpha}\frac{\partial}{\partial\tau^{\alpha}} + \int q_{\lambda}pd\lambda + i\int(N^a_{\lambda}\frac{\delta r_b}{\delta u} + N^a_{\lambda n}\frac{\delta r_b}{\delta u_n})d\lambda r_a\frac{\partial}{\partial r_b} +$$

$$+\int(N^a_n N^c_{\lambda} - N^a N^c_{\lambda n})d\lambda r_a r_c. \tag{11}$$

So in H_0 and P_0 enter addends depending on nonphysical variables. Those variables could be removed from dynamic scheme via appropriate choice of state vector on which commutators of integrals of motion are equal zero.

If we demand $F^2_{\lambda}\,|_{\partial c} = 0$, $F_{\lambda}F\,|_{\partial c} = 0$, $\int VF_{\lambda\lambda}d\lambda = 0$ и $\int VFd\lambda = 0$ then expressions for H_0 and P_0 look like:

$$H_0 = in^{\alpha}\frac{\partial}{\partial\tau^{\alpha}} + h_0, \qquad P_0 = ie^{\alpha}\frac{\partial}{\partial\tau^{\alpha}} + p_0,$$

where h_0 and p_0 are the quadratic forms with respect to r_a and canonical momenta.

Commutator of h_0 and p_0 is equal zero on the state vector which looks like

$$\Phi(\xi,\eta) = Cexp(\hat{\alpha}(\xi^2 - \eta^2)),$$

here ξ and η are linear combinations of r_a. So h_0 and p_0 can be removed from dynamical scheme.

Hence

$$H_0 = in^{\alpha}\frac{\partial}{\partial\tau^{\alpha}} = i\frac{\partial}{\partial\tau^1},$$

so

$$i\frac{\partial\Psi}{\partial t} = [\Psi, H], \qquad i\frac{\partial}{\partial t}\Psi = -i\frac{\partial}{\partial\tau^1}\Psi \Rightarrow \qquad \Psi(t) = \Psi(t-\tau^1).$$

Analogously for the momentum:

$$\Psi(x) = \Psi(x-\tau^2).$$

So energy and momentum conservation lows are accomplished.

5. CONCLUSION

Nonstationary equation describing polaron motion has been obtained in present work. It was possible after Bogoliubov group variables definition. To that end we have solved a more general problem of classical component separation in secondary quantized system.

Integrals of motion for the quantized neutral field are reduced to the derivatives with respect to group variables which turn out to be cyclic.

It permits to take into account conservation lows accurately within zero-point order with respect to parameter g.

Further development of perturbation theory scheme requires group variables specifying and additional conditions. However it demands only technical efforts, course in principle the accurate account of conservation lows problem is solved namely in orders considered already .

Note that proposed formalism permits to consider arbitrary motions of charged particle, for example periodic motion or particle motion in external field (group variables not cyclic in this case but this is not important for the formalism development).

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