

High Energy Physics in the 21-st Century

— Unified Supersymmetric Composite Model of All Fundamental Particles and Forces*

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Abstract

The unified supersymmetric composite model of all fundamental particles (and forces) including not only the fundamental fermions (quarks and leptons) but also the fundamental bosons (gauge bosons and Higgs scalars) is reviewed in detail. The contents include:

- I. Introduction
- II. Subquarks
- III. Weak Currents
- IV. Weak-Mixing Angle
- V. Quark-Mixing Matrix
- VI. Subquark, Quark and Lepton, Baryon, Weak-Boson, and Higgs Scalar Masses
- VII. “Baryon Generations”
- VIII. Leptoquarks and Other Exotics
- IX. Neutrino Masses and Mixings
- X. Conclusion, Further Discussions and Future Prospects.

Preface

In 1897, the electron was discovered by Thomson and in nineteen twenties the quantum theory of the electron or quantum electrodynamics was invented by Dirac. In nineteen sixties, the unified gauge theory of $SU(2)_w \times U(1)_Y$ for weak and electromagnetic (or electroweak) interactions of the electron (and all the other fundamental fermions, quarks and leptons,) or quantum “electroweakdynamics” was proposed by Glashow, Salam and Weinberg [1]. However, not only this unified theory but also its extension, the standard model of quarks and leptons including the Yang-Mills gauge theory of $SU(3)_c$, quantum chromodynamics [2], has so many arbitrary parameters that it may not be a unified (or final) theory but be an effective (or approximate) theory at low energies. In the middle of nineteen seventies, composite models of quarks and leptons were proposed by Pati and Salam and by ourselves [3, 4, 5, 6, 7]. In this talk, I am going to review the unified composite model of all “elementary” particles including not only the fundamental fermions (quarks and leptons) but also the fundamental bosons (gauge bosons and Higgs scalars).

The contents of this talk will include: Introduction in Section I, Subquarks in Section II, Weak Currents in Section III, Weak-Mixing Angle in Section IV, Quark-Mixing Matrix in Section V, Subquark, Quark and Lepton, Baryon, Weak-Boson, and Higgs Scalar Masses in Section VI, “Baryon Generations” in Section VII, Leptoquarks and Other Exotics in Section VIII, Neutrino Masses and Mixings in Section IX, and Conclusion, Further Discussions, and Future Prospects in Section X.

*The talk was actually made under the title: “Unified Supersymmetric Composite Model of All Fundamental Particles and Forces”.

Some parts of the contents of this talk will overlap those of Ref. [7] and even simulate those of my latest talks in Ref. [8].

I. Introduction

In January 1996, the CDF Collaboration at the Fermilab Tevatron collider [9] released their data on the inclusive jet differential cross section for jet transverse energies, E_T , from 15 to 440 GeV, in the pseudorapidity region $0.1 \leq |\eta| \leq 0.7$, with the significant excess over current predictions based on perturbative QCD calculations for $E_T > 200$ GeV, which may indicate the presence of quark substructure at the compositeness energy scale, Λ_C , of the order of 1.6 TeV. It can be taken as an exciting and already intriguing historical discovery of the substructure of quarks (and leptons), which has been long predicted, or as the first evidence for the composite model of quarks (and leptons), which has been long proposed since the middle of 1970's [3, 4, 5, 6, 7]. It may dramatically change not only the so-called “common sense” in physics or science but also that in philosophy, which often states that quarks (and leptons) are the smallest and most fundamental forms (or particles) of matter in the “mother nature”. Note that such relatively low energy scale for Λ_C of the order of 1 TeV has recently been anticipated rather theoretically [10] or by precise comparison between currently available experimental data and calculations in the composite model of quarks (and leptons) [11]. However, the experimental indication would certainly encourage us, “composite modelists”, to continue to study the composite model of quarks (and leptons) extensively and to make more predictions for future experimental tests of the model. In fact, in 1997, the H1 [12] and ZEUS [13] Collaborations at HERA reported their data on the deep-inelastic e^+p scattering with a significant excess of events over the expectation of the standard model of electroweak and strong interactions for high momentum-transfers squared $Q^2 > 15000 GeV^2$, which may indicate a sign for new physics beyond the standard model. Although neither of these indications have been confirmed and the significance of the HERA anomaly has decreased with higher statistics, not only the possible substructure of quarks and leptons as well as Higgs scalars and gauge bosons but also the possible existence of leptoquarks have been extensively re-investigated.

The purpose of this talk is to present in a systematic way many relations for the weak-mixing angle and the charges, weak-isospins, masses, and mixing-matrix elements of hadrons, quarks, subquarks, weak bosons, and Higgs scalars, some of which have previously been derived and some others are newly derived from “triplicity” (which asserts that a certain physical quantity such as the weak currents can be taken equally well as a composite operator of hadrons, of quarks, or of subquarks) [14]. From these “trinity” relations, we shall not only predict the weak-mixing angle, the masses of hadrons, quarks, subquarks, and Higgs scalars and the quark-mixing matrix elements, but also suggest the existence of “baryon generations” similar to quark-lepton generations.

Let us first transform the original notion of “triplicity” into the following more general notion of “trinity”: since hadrons are made of quarks and both quarks and leptons are supposed to be made of subquarks, all the matters can be taken equally well as composites of hadrons and leptons, of quarks and leptons, or of subquarks. Furthermore, since there are many similarities between composite dynamics of binding quarks into hadrons and that of binding subquarks into quarks and leptons, one can introduce many analogies into

subquark dynamics: in analogy of quantum chromodynamics (QCD) for strong interaction of quarks and gluons [2], quantum subchromodynamics (QSCD) for strong interaction of subquarks and “subgluons”, the Yang-Mills gauge theory of “subcolors” [15]; in analogy of a hadronic string [16] between a quark and an antiquark in a meson or between a quark and a (scalar) diquark in a baryon, a string or a superstring [17] between a subquark and an antisubquark in a gauge boson or a Higgs scalar or between a (spinor) subquark and a (scalar) subquark or a (scalar) disubquark in a quark or a lepton; and in analogy of supersymmetry for quarks and leptons [18, 19, 20], supersymmetry for subquarks [21] are all working hypotheses for subquark dynamics. The only difference between quark dynamics and subquark dynamics lies in the fact that the compositeness energy scale of quarks, Λ_C , may be of the order of 1 TeV, about 10^4 times larger than that of hadrons, Λ_s ($\cong 200$ MeV) [22]. Therefore, it seems very natural that a certain relation among the properties of hadrons derived from the quark model of hadrons can be transformed into an analogous relation among those of quarks and leptons to be derived from the subquark model of quarks and leptons, and that if the relation holds for hadrons, the analogous relation does for quarks and leptons, and vice versa. In Sections III-VII, I shall present and discuss such “trinity” relations one by one from this point of view.

II. Subquarks

The minimal supersymmetric composite model of quarks and leptons consists of an isodoublet of spinor subquarks with charges $\pm 1/2$, w_1 and w_2 (called “wakems” standing for weak and elemagnetic) [4], and a Pati-Salam color-quartet of scalar subquarks with charges $+1/2$ and $-1/6$, C_0 and C_i ($i = 1, 2, 3$) (called “chroms” standing for colors) [3]. The spinor and scalar subquarks with the same charge $+1/2$, w_1 and C_0 , may form a fundamental multiplet of $N = 1$ supersymmetry [21]. Also, all the six subquarks, w_i ($i = 1, 2$) and C_α ($\alpha = 0, 1, 2, 3$), may have “subcolors”, the additional degrees of freedom [15], and belong to a fundamental representation of subcolor symmetry. Although the subcolor symmetry, if any, is unknown, a simplest and most likely candidate for it is $SU(4)$. Therefore, for simplicity in this talk, all the subquarks are assumed to be quartet in subcolor $SU(4)$. Also, although the confining force between subquarks is unknown, a simplest and most likely candidate for it is the one described by quantum subchromodynamics (QSCD), the Yang-Mills gauge theory of subcolor $SU(4)$ [15]. The quantum numbers of these six subquarks are summarized in Table 1. Note that the subquark charges satisfy not only the Nishijima-Gell-Mann rule of $Q = I_w + (B - L)/2$ but also the “anomaly-free condition” of $\sum_w Q_w = \sum_C Q_C = 0$.

Table 1. The quantum numbers of the subquarks

name	charge Q	baryon no. B	lepton no. L	spin J	isospin $SU(2)_w$	color $SU(4)_c$	subcolor $SU(4)_{sc}$
w_1	$+1/2$	} 0	} 0	} $1/2$	$\underline{2}$	$\underline{1}$	$\underline{4}$
w_2	$-1/2$						
C_0	$+1/2$	0	-1	} 0	$\underline{1}$	$\underline{4}$	$\underline{4}$
C_i ($i=1,2,3$)	$-1/6$	$-1/3$	0				

In the minimal supersymmetric composite model, we expect that there exist at least 36 ($= 6 \times 6$) composite states of a subquark and an antisubquark which are subcolor-singlet. They include 1) 16($= 4 \times 2 \times 2$) spinor states corresponding to one generation of quarks and leptons, and their antiparticles of

$$\begin{aligned} \nu &= \bar{C}_0 w_1 & u_i &= \bar{C}_i w_1 & \bar{\nu} &= C_0 \bar{w}_1 & \bar{u}_i &= C_i \bar{w}_1 \\ l &= \bar{C}_0 w_2 & d_i &= \bar{C}_i w_2 & \bar{l} &= C_0 \bar{w}_2 & \bar{d}_i &= C_i \bar{w}_2 \end{aligned} \quad (i = 1, 2, 3),$$

2) 4($= 2 \times 2$) vector states corresponding to the photon and weak bosons of

$$W^+ = \bar{w}_2 w_1 \quad \gamma, Z = \bar{w}_1 w_1, \bar{w}_2 w_2, \bar{C}_0 C_0, \bar{C}_i C_i \quad W^- = \bar{w}_1 w_2$$

or 4($= 2 \times 2$) scalar states corresponding to the Higgs scalars of

$$\phi_i^j = \begin{pmatrix} \bar{w}_1 w_1 & \bar{w}_2 w_1 \\ \bar{w}_1 w_2 & \bar{w}_2 w_2 \end{pmatrix} \quad (i, j = 1, 2),$$

and 3) 16($= 4 \times 4$) vector states corresponding to a) the gluons, “leptogluon”, and “barygluon” of

$$G^a = \bar{C}_i \left(\frac{\lambda^a}{2} \right)_{ij} C_j \quad G^0 = \bar{C}_0 C_0 \quad G^9 = \bar{C}_i C_i \quad (i, j = 1, 2, 3),$$

where λ^a ($a = 1, 2, 3, \dots, 8$) is the Gell-Mann’s matrix of $SU(3)_c$, and b) the “vector leptoquarks” of

$$X_i = \bar{C}_0 C_i \quad \bar{X}_i = \bar{C}_i C_0 \quad (i = 1, 2, 3),$$

or 16($= 4 \times 4$) scalar states corresponding to the “scalar gluons”, “scalar leptogluon”, “scalar barygluon”, and “scalar leptoquarks” of

$$\Phi_\beta^\alpha = \bar{C}_\alpha C_\beta \quad (\alpha, \beta = 0, 1, 2, 3)$$

These 36 composite states are summarized in Table 2. Quarks or leptons with the same quantum numbers but in different generations can be taken as dynamically different composite states of the same constituents which I shall discuss in Sections V, VI, and VII.

Table 2. The composite states of a subquark and an antisubquark				
	\bar{w}_1	\bar{w}_2	\bar{C}_0	\bar{C}_j ($j = 1, 2, 3$)
w_1	$A_1^1(\phi_1^1)$	$W^+(\phi^+)$	ν	u_j
w_2	$W^-(\phi^-)$	$A_2^2(\phi_2^2)$	l	d_j
C_0	$\bar{\nu}$	\bar{l}	$G_0^0(\Phi_0^0)$	$G^{j+2/3}(\Phi^{j+2/3})$
C_i ($i = 1, 2, 3$)	\bar{u}_i	\bar{d}_i	$G_i^{-2/3}(\Phi_i^{-2/3})$	$G_i^j(\Phi_i^j)$

In addition to these “meson-like composite states” of a subquark and an antisubquark, there may also exist “baryon-like composite states” of 4 subquarks which are subcolor singlet. I shall discuss these states as exotic states in Section VIII.

III. Weak Currents

The weak charged current, J_μ , provides one of the most instructive examples of the physical quantities to which triplicity or trinity of hadrons, quarks, and subquarks can be applied. It can be written in terms of hadrons (baryons and mesons) and leptons as a sum of so many phenomenological terms,

$$J_\mu \cong \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu + \bar{\nu}_\tau \gamma_\mu (1 - \gamma_5) \tau + \frac{G^\beta}{G^\mu} \bar{p} \gamma_\mu \left(1 - \frac{g_A^\beta}{g_V^\beta} \gamma_5 \right) n + \frac{G^\Lambda}{G^\mu} \bar{p} \gamma_\mu \left(1 - \frac{g_A^\Lambda}{g_V^\Lambda} \gamma_5 \right) \Lambda + \dots, \quad (1)$$

where G^β/G^μ , g_A^β/g_V^β , \dots are phenomenological parameters. It can also be written in terms of quarks, $U_i = (u_i, c_i, t_i)$ and $D_i = (d_i, s_i, b_i)$ ($i = 1, 2, 3$), and leptons, $N = (\nu_e, \nu_\mu, \nu_\tau)$ and $L = (e, \mu, \tau)$, as the sum of at least twelve terms,

$$J_\mu \cong \bar{N} \gamma_\mu (1 - \gamma_5) L + \bar{U}_i \gamma_\mu (1 - \gamma_5) V D_i = \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu + \bar{\nu}_\tau \gamma_\mu (1 - \gamma_5) \tau + V_{ud} \bar{u}_i \gamma_\mu (1 - \gamma_5) d_i + V_{us} \bar{u}_i \gamma_\mu (1 - \gamma_5) s_i + \dots, \quad i = 1, 2, 3, \quad (2)$$

where $V_{UD}(= V_{ud}, V_{us}, \dots)$ are the nine parameters called CKM quark-mixing matrix elements [23]. Note that no lepton-mixing (i.e., $U = 1$, where U is the lepton-mixing matrix) and the only three generations of quarks and leptons are assumed for simplicity unless otherwise stated in this talk. Furthermore, it is now well-known that in the minimal composite model of quarks and leptons [3, 4, 5, 6, 7], it can be most simply written in terms of the wakems w_1 and w_2 , as a single term without any free parameters,

$$J_\mu = \bar{w}_1 \gamma_\mu (1 - \gamma_5) w_2. \quad (3)$$

IV. Weak-Mixing Angle

In the unified subquark model of quarks and leptons [3, 4, 5, 6, 7], not only quarks and leptons but also gauge bosons such as the weak bosons (W^\pm and Z), the photon (A), and the gluons (G^a , $a = 1 - 8$) can be taken as composite states of subquarks,

$$W_\mu^+ = \bar{w}_{2L} \gamma_\mu w_{1L}, \quad W_\mu^- = \bar{w}_{1L} \gamma_\mu w_{2L}, \quad (4)$$

$$A_\mu = \frac{\sqrt{3}}{2} \left(\frac{1}{2} \bar{w}_1 \gamma_\mu w_1 - \frac{1}{2} \bar{w}_2 \gamma_\mu w_2 + \frac{1}{2} i C_0^\dagger \overleftrightarrow{\partial}_0 C_0 - \frac{1}{6} i C_i^\dagger \overleftrightarrow{\partial}_\mu C_i \right) (\equiv \sin \theta_w A_\mu^3 + \cos \theta_w B_\mu), \quad (5)$$

$$Z_\mu = \frac{\sqrt{5}}{2} \left(\frac{1}{2} \bar{w}_{1L} \gamma_\mu w_{1L} - \frac{1}{2} \bar{w}_{2L} \gamma_\mu w_{2L} \right) - \frac{3\sqrt{5}}{10} \left(\frac{1}{2} \bar{w}_{1R} \gamma_\mu w_{1R} - \frac{1}{2} \bar{w}_{2R} \gamma_\mu w_{2R} + \frac{1}{2} i C_0^\dagger \overleftrightarrow{\partial}_\mu C_0 - \frac{1}{6} i C_i^\dagger \overleftrightarrow{\partial}_\mu C_i \right) (\equiv \cos \theta_w A_\mu^3 - \sin \theta_w B_\mu), \quad (6)$$

$$G_\mu^a = i\sqrt{2}C_i^\dagger \overleftrightarrow{\partial}_\mu \left(\frac{\lambda^a}{2} \right)_{ij} C_j, \quad (7)$$

where A_μ^3 and B_μ are the third components of the iso-triplet gauge bosons and the iso-scalar gauge boson in the Glashow-Salam-Weinberg $SU(2)_L \times U(1)_Y$ unified gauge theory of electroweak interactions [1], and θ_w is the weak-mixing angle. These relations can be taken either as those derived from our unified subquark model of the Nambu-Jona-Lasinio type [4, 5, 6, 7, 24] or as field-current identities [25] for the gauge fields and subquark currents. In either way, it is now an elementary exercise to derive the Georgi-Glashow relations [26],

$$\sin^2 \theta_w = \Sigma(I_3)^2 / \Sigma Q^2 = \frac{3}{8} \quad (8)$$

and

$$f^2/g^2 = \Sigma(I_3)^2 / \Sigma(\lambda^a/2)^2 = 1 \quad (9)$$

for the gluon and weak boson coupling constants (f and g), the third component of the isospin (I_3), the charge (Q), and the color-spin ($\lambda^a/2$) of subquarks from the relations (5)-(7), without depending on the assumption of grand unification of strong and electroweak interactions.

Similarly, in our unified quark-lepton model of the Nambu-Jona-Lasinio type [4, 5, 6, 7, 24] or in the field-current identification for gauge fields and quark-lepton currents, the gauge boson fields can be taken, at least approximately, as composite operators made of quarks and leptons,

$$\begin{aligned} W_\mu^+ &\cong \frac{1}{\sqrt{4N_g}} (\bar{e}_L \gamma_\mu \nu_{eL} + \bar{d}_{iL} \gamma_\mu u_{iL} + \dots), \\ W_\mu^- &\cong \frac{1}{\sqrt{4N_g}} (\bar{\nu}_{eL} \gamma_\mu e_L + \bar{u}_{iL} \gamma_\mu d_{iL} + \dots), \end{aligned} \quad (10)$$

$$A_\mu \cong \frac{\sqrt{3}}{4\sqrt{N_g}} \left(-\bar{e} \gamma_\mu e + \frac{2}{3} \bar{u}_i \gamma_\mu u_i - \frac{1}{3} \bar{d}_i \gamma_\mu d_i + \dots \right), \quad (11)$$

$$\begin{aligned} Z_\mu &\cong \frac{\sqrt{5}}{4\sqrt{N_g}} \left(\frac{1}{2} \bar{\nu}_{eL} \gamma_\mu \nu_{eL} - \frac{1}{2} \bar{e}_L \gamma_\mu e_L + \frac{1}{2} \bar{u}_{iL} \gamma_\mu u_{iL} - \frac{1}{2} \bar{d}_{iL} \gamma_\mu d_{iL} + \dots \right) \\ &\quad - \frac{3\sqrt{5}}{20\sqrt{N_g}} \left(-\frac{1}{2} \bar{\nu}_{eL} \gamma_\mu \nu_{eL} - \frac{1}{2} \bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R \right. \\ &\quad \left. + \frac{1}{6} \bar{u}_{iL} \gamma_\mu u_{iL} + \frac{1}{6} \bar{d}_{iL} \gamma_\mu d_{iL} + \frac{2}{3} \bar{u}_{iR} \gamma_\mu u_{iR} - \frac{1}{3} \bar{d}_{iR} \gamma_\mu d_{iR} + \dots \right), \end{aligned} \quad (12)$$

$$G_\mu^a \cong \frac{1}{2\sqrt{N_g}} \bar{u}_i \gamma_\mu \left(\frac{\lambda^a}{2} \right)_{ij} u_j + \dots, \quad (13)$$

where N_g is the number of generations (≥ 3). It is almost trivial to derive the Georgi-Glashow relations (8) and (9) from these approximate identities.

Furthermore, all these gauge bosons, except the gluons, can also be taken as composite operators made of hadrons (baryons and mesons). By ignoring not only quark mixing, but also all hadrons other than the ground-state baryons of spin 1/2 and weak-isospin 1/2, they can be most roughly written as

$$\begin{aligned} W_\mu^+ &\cong \frac{1}{\sqrt{2N_g}}(\bar{e}_L\gamma_\mu\nu_{eL} + \bar{n}_L\gamma_\mu p_L + \dots), \\ W_\mu^- &\cong \frac{1}{\sqrt{2N_g}}(\bar{\nu}_{eL}\gamma_\mu e_L + \bar{p}_L\gamma_\mu n_L + \dots), \end{aligned} \quad (14)$$

$$A_\mu \cong \frac{1}{2\sqrt{N_g}}(-\bar{e}\gamma_\mu e + \bar{p}\gamma_\mu p + \dots), \quad (15)$$

and

$$\begin{aligned} Z_\mu &\cong \frac{\sqrt{3}}{4\sqrt{N_g}}(\bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \bar{e}_L\gamma_\mu e_L + \dots) \\ &\quad - \frac{\sqrt{3}}{12\sqrt{N_g}}(-\bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \bar{e}_L\gamma_\mu e_L - 2\bar{e}_R\gamma_\mu e_R + \bar{p}_L\gamma_\mu p_L \\ &\quad + \bar{n}_L\gamma_\mu n_L + 2\bar{p}_R\gamma_\mu p_R + \dots). \end{aligned} \quad (16)$$

It is again trivial to derive the following Georgi-Glashow relation from these very rough identities,

$$\sin^2 \theta_w = \Sigma(I_3)^2 / \Sigma Q^2 = \frac{1}{4}. \quad (17)$$

The numerical result for the weak-mixing angle in the subquark picture remarkably coincides with that in the quark picture (as in (8)) but differs from that in the ‘‘hadron picture’’ of (17). This coincidence (or ‘‘duality’’) seems more than a mere coincidence as it is caused by the same degrees of freedom due to the four wakems ($w_{1L}, w_{2L}, w_{1R}, w_{2R}$) and four chroms (C_0, C_1, C_2, C_3) forming a ‘‘subquark-superquartet’’. The experimental value is $\sin^2 \theta_w [\equiv 1 - (m_W^2/m_Z^2)] = 0.2224 \pm 0.0006$ for $m_W = 80.41 \pm 0.10$ GeV and $m_Z = 91.187 \pm 0.007$ GeV [27]. The disagreement between the value of 3/8 predicted either in the subquark model or in the quark model and the experimental value might be excused by insisting that the predicted value is viable as the running value renormalized à la Georgi, Quinn and Weinberg [28] at extremely high energies (as high as 10^{15} GeV), given the ‘‘desert hypothesis’’. On the other hand, it is more comfortable to find that the value of 1/4 obtained in the hadron picture agrees remarkably well with the experimental value. To sum up, triplicity or trinity of hadrons, quarks, and subquarks predicts that the quark-mixing angle increases as momentum-transfers go up from $\sin^2 \theta_w \cong 1/4$ at the energy scale of hadrons ($\Lambda_s \cong 200$ MeV) to $\sin^2 \theta_w \cong 3/8$ at the energy scales of quarks and subquarks ($\Lambda_C \gtrsim 1$ TeV), which simulates the old prediction by Georgi, Quinn and Weinberg [28] in the Georgi-Glashow $SU(5)$ grand-unified gauge theory of strong and electroweak interactions [26] although our predictions are not based on their hypothesis of grand unification.

V. Quark-Mixing Matrix

As the “hadron-quark duality” relations of (1) and (2) for the weak charged current mass-produce the approximate relations

$$\begin{aligned} \frac{G^\beta}{G^\mu} \bar{p} \gamma_\mu \left(1 - \frac{g_A^\beta}{g_V^\beta} \gamma_5 \right) n &\cong V_{ud} \langle p | \bar{u} \gamma_\mu (1 - \gamma_5) d | n \rangle, \\ \frac{G^\Lambda}{G^\mu} \bar{p} \gamma_\mu \left(1 - \frac{g_A^\Lambda}{g_V^\Lambda} \right) \Lambda &\cong V_{us} \langle p | \bar{u} \gamma_\mu (1 - \gamma_5) s | \Lambda \rangle, \dots, \end{aligned} \quad (18)$$

so do the “quark-subquark duality” relations of (2) and (3) the following [29]:

$$\begin{aligned} V_{ud} \bar{u} \gamma_\mu (1 - \gamma_5) d &\cong \langle u | \bar{w}_1 \gamma_\mu (1 - \gamma_5) w_2 | d \rangle, \\ V_{us} \bar{u} \gamma_\mu (1 - \gamma_5) s &\cong \langle u | \bar{w}_1 \gamma_\mu (1 - \gamma_5) w_2 | s \rangle, \dots \end{aligned} \quad (19)$$

By using the algebra of subquark currents [5], the unitarity of quark mixing matrix, $VV^+ = V^+V = 1$, has been demonstrated [30] although the superficial non-unitarity of V as a possible evidence for the substructure of quarks has very lately been discussed in detail by myself [31] just before the possible evidence released by the CDF Collaboration [9].

In the first-order perturbation of isospin breaking (the Hamiltonian H_I), the relations

$$\begin{aligned} V_{us} &= \frac{\langle u | H_I | c \rangle}{m_u - m_c} + \frac{\langle d | H_I | s \rangle}{m_s - m_d}, \quad V_{cd} = \frac{\langle c | H_I | u \rangle}{m_c - m_u} + \frac{\langle s | H_I | d \rangle}{m_d - m_s} \\ V_{cb} &= \frac{\langle c | H_I | t \rangle}{m_c - m_t} + \frac{\langle s | H_I | b \rangle}{m_b - m_s}, \quad V_{ts} = \frac{\langle t | H_I | c \rangle}{m_t - m_c} + \frac{\langle b | H_I | s \rangle}{m_s - m_b}, \dots \end{aligned} \quad (20)$$

have been obtained. From these follow immediately the anti-symmetry relations

$$V_{us} = -V_{cd}^*, \quad V_{cb} = -V_{ts}^*, \dots, \quad (21)$$

which agree well with the experimental values of $|V_{us}| = 0.217 \sim 0.224$ and $|V_{cd}| = (0.217 \sim 0.224)$ [27]. They also produce some other relations such as

$$|V_{cb}| (= |V_{ts}|) \cong (m_s/m_b) |V_{us}| \cong 0.021, \quad (22)$$

which roughly agrees with the latest experimental value of $|V_{cb}| = 0.036 \sim 0.042$ [27].

In the second-order perturbation, the relations

$$|V_{ub}| \cong (m_s/m_c) |V_{us} V_{cb}| \cong 0.0017 \quad (23)$$

and

$$|V_{td}| \cong |V_{us} V_{cb}| \cong 0.0046 \quad (24)$$

have been predicted. The relation (23) agrees remarkably well with the latest experimental data $|V_{ub}| = 0.0018 \sim 0.0045$ [27]. The predictions (22) for $|V_{ts}|$ and (24) for $|V_{td}|$

also agree fairly well with the experimental estimates from the assumed unitarity of V , $|V_{ts}| = 0.035 \sim 0.042$ and $|V_{td}| = 0.004 \sim 0.013$ [27]. It is, however, highly desirable to test the predictions more directly in the future experiments of top-quark decays.

To sum up, we have obtained the following prediction [31] of

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.975 & 0.2205 \pm 0.0018 & 0.0017 \\ \text{\small (0.9745~0.9760)} & \text{\small (input)} & \text{\small (0.0018~0.0045)} \\ 0.2205 \pm 0.0018 & 0.975 & 0.021 \\ \text{\small (0.217~0.224)} & \text{\small (0.9737~0.9753)} & \text{\small (0.036~0.042)} \\ 0.0046 & 0.021 & 0.9996 \\ \text{\small (0.004~0.013)} & \text{\small (0.035~0.042)} & \text{\small (0.9991~0.9994)} \end{pmatrix} \quad (25)$$

where the values indicated in the parentheses denote the experimental [27], to which our predicted values should be compared. Note that the theoretical values for V_{ud} , V_{cs} , and V_{tb} and the experimental ones for V_{cs} , V_{td} , V_{ts} , and V_{tb} , are all estimates from the assumed unitarity of V , which is subject to doubt in Ref. [32].

VI. Subquark, Quark and Lepton, Baryon, Weak-Boson and Higgs Scalar Masses

The unified subquark model predicts [3, 4, 5, 6, 7] the following two sum rules,

$$m_W = [3(m_{w_1}^2 + m_{w_2}^2)/2]^{1/2} \quad (26)$$

and

$$m_H = 2[(m_{w_1}^4 + m_{w_2}^4)/(m_{w_1}^2 + m_{w_2}^2)]^{1/2}, \quad (27)$$

where m_H is the mass of the physical Higgs scalar in the Glashow-Salam-Weinberg theory. Also, the unified quark-lepton model of the Nambu-Jona-Lasinio type [4, 5, 6, 7, 24] predicts the following two sum rules,

$$m_W = [3\langle m_{q,l}^2 \rangle]^{1/2} \quad (28)$$

and

$$m_H = 2[\sum m_{q,l}^4 / \sum m_{q,l}^2]^{1/2}, \quad (29)$$

where $m_{q,l}$'s are the quark and lepton masses and $\langle \rangle$ denotes the average value for all the quarks and leptons. Notice that the second sum rules (27) and (29) are essentially the same as the Nambu relation [33] $m_\xi : m_\psi : m_\eta = 0 : 1 : 2$ or $m_\xi^2 + m_\eta^2 = 4m_\psi^2$, where ξ , η , and ψ are the Nambu-Goldstone boson, the physical scalar, and the constituent fermion, respectively, and that they are the consequences of Nambu's supersymmetry and, therefore, less model-dependent.

By combining the sum rules (26) and (27), the following relation can be obtained for $m_{w_1} = m_{w_2} = m_w$,

$$m_w : m_W : m_H = 1 : \sqrt{3} : 2. \quad (30)$$

From this relation, the wakem and Higgs scalar masses can be predicted as

$$m_w = (1/\sqrt{3})m_W = (46.4 \pm 0.1) \text{ GeV} \quad (31)$$

and

$$m_H = (2/\sqrt{3})m_W = (92.8 \pm 0.1) \text{ GeV} \quad \text{for } m_W = (80.41 \pm 0.10) \text{ GeV} , \quad (32)$$

which is subject to a future experimental test, probably at LEP II. More precisely, from the two sum rules the Higgs mass can be bounded as

$$(92.8 \pm 0.1) \text{ GeV} = (2/\sqrt{3})m_W \leq m_H \leq (2\sqrt{6}/3)m_W = (131.3 \pm 0.2) \text{ GeV} . \quad (33)$$

Notice that the lower bound corresponds to the case of $m_{w_1} = m_{w_2}$ while the upper one to that of $m_{w_1}/m_{w_2} = 0$ or ∞ . Therefore, it seems more likely that the physical Higgs scalar will be found close to the lower bound, *i.e.*, $m_H \cong 93 \text{ GeV}$. The reliability of this prediction may be enhanced by the following independent observation. Suppose that the subquark dynamics is described by QSCD [15]. Then, the masses of W^\pm and H are scaled by Λ_{sc} , the energy scale of QSCD, while the masses of the corresponding hadrons, ρ^\pm and σ , are scaled by Λ_s , the energy scale of QCD. If this is the case, the Higgs scalar mass can be estimated as

$$m_H \cong \frac{m_\sigma}{m_\rho} m_W \cong \frac{\sim 900 \text{ MeV}}{770 \text{ MeV}} (80.41 \pm 0.10) \text{ GeV} \cong 94 \text{ GeV} , \quad (34)$$

which amazingly produces a similar prediction, $m_H \cong 93 \text{ GeV}$.

If there exist only three generations of quarks and leptons, the sum rules (28) and (29) completely determine the top quark and Higgs scalar masses as

$$m_t \cong (2\sqrt{6}/3)m_W = (131.3 \pm 0.2) \text{ GeV} \quad (35)$$

and

$$m_H \cong 2m_t \cong (4\sqrt{6}/3)m_W = (262.6 \pm 0.3) \text{ GeV} \\ \text{for } m_W = (80.41 \pm 0.10) \text{ GeV} . \quad (36)$$

If there are four generations, the sum rule (28) would give and estimate for the average mass of the fourth generation of quarks and leptons as

$$[\langle m_{q,l}^2 \rangle_{N_g=4}]^{1/2} \cong (2/\sqrt{3}) m_W = (92.8 \pm 0.1) \text{ GeV} \\ \text{for } m_W = (80.41 \pm 0.10) \text{ GeV} . \quad (37)$$

Triplicity or trinity of hadrons, quarks, and subquarks tells us that these sum rules can be further extended to the approximate sum rules of

$$m_W \cong [3\langle m_{B,l}^2 \rangle]^{1/2} \quad (38)$$

and

$$m_H \cong 2[\sum m_{B,l}^4 / \sum m_{B,l}^2]^{1/2}, \quad (39)$$

where $m_{B,l}$'s are the “canonical baryon” and lepton masses and $\langle \rangle$ denotes the average value for all the canonical baryons and leptons. The “canonical baryon” means either one of p , n and other ground-state baryons of spin 1/2 and weak-isospin 1/2 consisting of a quark heavier than the u and d quarks and a scalar and isoscalar diquark made of u and d quarks. These sum rules can be derived, in the same way as for (26)-(29), in the “unified hadron-lepton model” of the Nambu-Jona-Lasinio type which is written in terms of the canonical baryons and leptons as fundamental fermions.

If there exist only three generations of quarks and leptons, the sum rules (38) and (39) completely determine the masses of the canonical topped baryon, T , and the Higgs scalar as respectively

$$m_T \cong 2m_W = (160.8 \pm 0.2) \text{ GeV} \quad (40)$$

and

$$m_H \cong 2m_T \cong 4m_W = (321.6 \pm 0.4) \text{ GeV} \quad \text{for } m_W = (80.41 \pm 0.10) \text{ GeV} . \quad (41)$$

If, instead, there are four generations, the sum rule (38) gives an estimate for the average mass of the fourth generation of the canonical baryons and leptons the same as in (37). Notice that the predicted value for the canonical topped baryon mass in (40) is 22 percent larger than that for the top quark mass in (35) and that the predicted values for the Higgs scalar mass in (32), (36) and (40) are in the ratio $1 : 2\sqrt{2} : 2\sqrt{3}$. Particularly the latter may indicate either that there exist at least four generations of quarks and leptons or that both the unified quark-lepton models of the Nambu-Jona-Lasinio type and the unified hadron-lepton type are poor approximations for describing the Higgs scalar. The answer may be given by future high-energy experiments.

In the remaining part of this Section, I shall present more recent progress in predicting the quark, lepton, and Higgs scalar masses in the minimal supersymmetric composite model of quarks and leptons. The notion of almost Nambu-Goldstone fermions has first been introduced by myself [21] into the minimal composite model of quarks and leptons [3, 4, 5, 6, 7] in which quarks and leptons consist of a wakem (w_i for $i = 1$ or 2) and a chrom (C_α for $\alpha = 0, 1, 2$, or 3). By taking the first generation of quarks and leptons as almost Nambu-Goldstone fermions due to spontaneous breakdown of approximate supersymmetry between a wakem and a chrom, and the second generation of them as quasi Nambu-Goldstone fermions [34, 35, 36], the superpartners of Nambu-Goldstone bosons due to spontaneous breakdown of approximate global symmetry, we have not only explained the hierarchy of quark and lepton masses, $m_e \ll m_\mu \ll m_\tau$, $m_u \ll m_c \ll m_t$, and $m_d \ll m_s \ll m_b$ (if the constituent subquark mass scale, $M_{w,C}$, is much smaller than the compositeness energy scale of quarks and leptons, Λ_C , since the masses of first, second, and third generations of quarks and leptons are of the orders of $M_{w,C}^3/\Lambda_C^2$, $M_{w,C}$, and Λ_C), but also derived the square-root sum rules for quark and lepton masses [37, 38],

$$m_e^{1/2} = m_d^{1/2} - m_u^{1/2}, \quad (42)$$

$$m_\mu^{1/2} - m_e^{1/2} = m_s^{1/2} - m_d^{1/2}, \quad (43)$$

and the simple relations among quark and lepton masses [39, 40],

$$m_e m_\tau^2 \cong m_\mu^3, \quad (44)$$

$$m_u m_s^3 m_t^2 \cong m_d m_c^3 m_b^2, \quad (45)$$

all of which are remarkably well satisfied by the experimental values and estimates.

By solving a set of these two sum rules and two relations (42)-(45) [41], we can obtain the following prediction [7]:

$$\begin{pmatrix} m_e & m_\mu & m_\tau \\ m_u & m_c & m_t \\ m_d & m_s & m_b \end{pmatrix} \cong \begin{pmatrix} 0.511 \text{ MeV} & 105.7 \text{ MeV} & 1520 \text{ MeV} \\ \text{(input)} & \text{(input)} & (1777.05^{+0.29}_{-0.26} \text{ MeV}) \\ 4.5 \pm 1.4 \text{ MeV} & 1350 \pm 50 \text{ MeV} & 183 \pm 78 \text{ GeV} \\ \text{(input)} & \text{(input)} & (173.8 \pm 5.2 \text{ GeV}) \\ 8.0 \pm 1.9 \text{ MeV} & 154 \pm 8 \text{ MeV} & 5.3 \pm 0.1 \text{ GeV} \\ (7.9 \pm 2.4 \text{ MeV}) & (155 \pm 50 \text{ MeV}) & \text{(input)} \end{pmatrix} \quad (46)$$

where the “inputs” and the values indicated in the parentheses denote either the experimental data [27, 42] or the phenomenological estimates [43], to which our predicted values should be compared. Furthermore, if we solve a set of the four sum rules, (28), (29), (42), and (43), and the two relations, (44) and (45), we can predict not only the four quark and/or lepton masses such as m_d , m_s , m_t , and m_τ in Eq. (46) but also the Higgs scalar and weak boson masses as

$$m_H \cong 2m_t = 366 \pm 156 \text{ GeV} , \quad (47)$$

$$m_W \cong \sqrt{3/8}m_t = 112 \pm 24 \text{ GeV} , \quad (48)$$

which should be compared to the experimental value of $m_W = 80.41 \pm 0.10 \text{ GeV}$ [27]. To sum up, we have succeeded in explaining and predicting most of the properties (masses and mixing angles) of quarks and leptons in the minimal supersymmetric composite model since we have explained the CKM quark-mixing matrix elements earlier as reviewed in the previous Section. Note also that I have very recently proposed a new model of baryons in which baryons are taken as almost Nambu-Goldstone fermions due to spontaneous breakdown of approximate supersymmetry between a constituent quark and a scalar diquark [44]. This model of baryons and its amazing consequences will be discussed in the following Section.

VII. “Baryon Generations”

A remarkable similarity or “duality” between baryons in the quark model of hadrons and quarks (and leptons) in the subquark model of quarks (and leptons) tempts us to imagine that there may exist “baryon generations” similar to quark-lepton generations. In particular, if the first and second generations of quarks and leptons are indeed taken as

almost and quasi Nambu-Goldstone fermions as in the minimal supersymmetric composite model of quarks and leptons and if the ground states of baryons can be taken as almost Nambu-Goldstone fermions as in the new supersymmetric model of baryons, it seems even natural to expect that there may exist the “second generation” of baryons which can be taken as quasi Nambu-Goldstone fermions. In addition, there may be the “third generation” of baryons which are “ordinary” composite states of a quark and a scalar diquark and which can be taken neither as almost Nambu-Goldstone composite fermions nor as quasi Nambu-Goldstone composite ones. Therefore, “duality” of baryons and quarks (and leptons) strongly suggests that there may exist three generations of baryons as three generations of quarks (and leptons) and that the square-root sum rules and simple relations similar to those for quark and lepton masses may also hold for baryon masses. Note, however, that there may exist no hierarchy in the mass spectrum of the first, second, and third generations of baryons as in that of quarks and leptons. This is because their masses of the orders of M_q^3/Λ_s^2 , M_q , and Λ_s are of the same order of magnitude since the constituent quark and diquark mass scale, M_q ($\cong 300 \sim 600$ MeV), is of the same order of magnitude as the compositeness energy scale of baryons, Λ_s ($\cong 200$ MeV).

The first square-root sum rule for quark and lepton masses is transformed into the ones such as

$$\begin{aligned} m_n^{1/2} - m_p^{1/2} &= m_{\Xi^-}^{1/2} - m_{\Xi^0}^{1/2} = m_{\Xi_c^0}^{1/2} - m_{\Xi_c^+}^{1/2} = \dots, \\ (0.021 \text{ MeV}^{1/2}) & \quad (0.088 \text{ MeV}^{1/2}) \quad (0.047 \text{ MeV}^{1/2}) \end{aligned} \quad (49)$$

which are not so well satisfied by the experimental values [27] indicated in the parentheses. On the other hand, the second square-root sum rule for quark and lepton masses is transformed into the ones such as

$$\begin{aligned} m_{N_2}^{1/2} - m_N^{1/2} &= m_{\Lambda_2}^{1/2} - m_\Lambda^{1/2} = m_{\Sigma_2}^{1/2} - m_\Sigma^{1/2} \\ (7.17 \sim 7.71 \text{ MeV}^{1/2}) & \quad (6.10 \sim 7.83 \text{ MeV}^{1/2}) \quad (5.77 \sim 6.58 \text{ MeV}^{1/2}) \\ &= m_{\Xi_2}^{1/2} - m_\Xi^{1/2} = \dots, \\ & \quad (7.6 \sim 8.1 \text{ MeV}^{1/2}) \end{aligned} \quad (50)$$

which are fairly well satisfied by the experimental values [27] indicated in the parentheses if $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, and $\Xi(1950)$ are identified with N_2 , Λ_2 , Σ_2 , and Ξ_2 baryons of the second generation. Also, the simple relations among quark and lepton masses is transformed into the ones such as

$$\begin{aligned} m_{N_3} &\cong (m_{N_2}^3/m_N)^{1/2} & m_{\Lambda_3} &\cong (m_{\Lambda_2}^3/m_\Lambda)^{1/2} \\ (1680 \sim 1740 \text{ MeV}) & \quad (1765 \sim 1839 \text{ MeV}) & (1750 \sim 1850 \text{ MeV}) & \quad (1845 \sim 2098 \text{ MeV}) \\ m_{\Sigma_3} &\cong (m_{\Sigma_2}^3/m_\Sigma)^{1/2} & m_{\Xi_3} &\cong (m_{\Xi_2}^3/m_\Xi)^{1/2} \quad \dots \\ (\approx 1880 \text{ MeV}) & \quad (1905 \sim 2011 \text{ MeV}) & (\approx 2250 \text{ MeV}) & \quad (2344 \sim 2399 \text{ MeV}) \end{aligned} \quad (51)$$

all of which are fairly well satisfied by the experimental values [27] if $N(1710)$, $\Lambda(1810)$, $\Sigma(1880)$, and $\Xi(2250)$ are identified with N_3 , Λ_3 , Σ_3 , and Ξ_3 baryons of the third generation.

In view of this successful identification of $N(1440)P_{11}$, $N(1710)P_{11}$, $\Lambda(1600)P_{01}$, $\Lambda(1810)P_{01}$, $\Sigma(1660)P_{11}$, $\Sigma(1880)P_{11}$, $\Xi(1950)$, and $\Xi(2250)$ with N_2 , N_3 , Λ_2 , Λ_3 , Σ_2 , Σ_3 , Ξ_2 , and Ξ_3 , respectively, we suggest that there exist at least three generations of baryons whose second and third generations are not ordinary (rotationally or radially) excited composite states of quarks but extraordinary composite states of quarks dynamically

different from their ground states of the first generation. This may solve the famous puzzle of the ‘‘Roper resonance’’, which has been long standing for the last more than three decades, *i.e.*, ‘‘What is $N(1440)P_{11}$?’’ or ‘‘How is $N(1440)P_{11}$ different from N ?’’. Before concluding this Section, I also wish to urge high-energy experimentalists to try to search for another (fourth) NP_{11} at the mass larger than 1710 MeV (or to confirm the candidate of $N(2100)P_{11}$) since the possible discovery of such baryon would indicate not only the existence of more than three generations of baryons but also that of more than three generations of quarks and leptons!

VIII. Leptoquarks and Other Exotics

In the previous Sections, I have discussed only specific composite states of a subquark and an antishquark corresponding to familiar fundamental particles such as quarks, leptons, gauge bosons, and Higgs scalars. The remaining composite states of a subquark and an antishquark correspond to the leptogluon, barygluon, and leptoquarks of

$$G^0 = \bar{C}_0 C_0, \quad G^9 = \bar{C}_i C_i, \quad \text{and} \quad X_i = \bar{C}_0 C_i (\bar{X}_i = \bar{C}_i C_0) \quad (i = 1, 2, 3)$$

or of

$$\Phi_\beta^\alpha = \bar{C}_\alpha C_\beta \quad (\alpha, \beta = 0, 1, 2, 3).$$

Since 1980, the possible existence of these exotic bosons has been predicted by myself [5] in the unified composite model of quarks and leptons. Very lately, Akama, Katsuura, and I [45] have found that possible production of ‘‘excited’’ bosons (‘‘excited’’ gluons, weak bosons, Higgs scalars, *etc.*) whose masses are around 2 TeV may be responsible for the CDF anomaly [9] while that of either leptoquarks whose masses are around 300 GeV or excited positrons whose masses are around 350 GeV may be responsible for the HERA anomaly [12, 13], and that these two anomalies can be explained by different ‘‘sectors’’ of the same model, the unified composite model of not only quarks and leptons, but also gauge bosons and Higgs scalars [4, 5, 6, 7]. Therefore, here I am happy to announce that the barygluon and leptoquarks might have been found by the CDF Collaboration and by the H1 and ZEUS Collaborations, respectively. Then, what would be left to be found is the leptogluon consisting of C_0 and \bar{C}_0 !

Also for a long time, I have predicted the possible existence of many ‘‘baryonic-like’’ composite states of subquarks such as $(w_1 w_1 w_1 w_1), \dots, (C_0 C_1 C_2 C_3)$, which are subcolor singlet. If subquarks are not permanently confined in subcolor-singlet states such as quarks, leptons, gauge bosons, Higgs scalars, leptoquarks, *etc.*, their fractional charges of $\pm 1/2$ and $\pm 1/6$ can be an excellent sign of their presence. Even if they are permanently confined but if the subcolor symmetry were not $SU(4)_{sc}$ but $SU(3)_{sc}$, there might exist subcolor-singlet and color-singlet states such as $(w_1 w_1 w_1), \dots, (C_1 C_2 C_3)$, *etc.* with the fractional charges of $\pm 3/2$ and $\pm 1/2$. Therefore, a possible discovery of the half-integer-charged particles would be a dramatic sign of subquarks. Such a search for half-charged particles had been performed. In the underground search by Orito *et al.* [46], no anomalous candidate was found and the upper bound on the flux of $1.6 \times 10^{-12} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ was reported. In fact, the fact that there may not exist the half-integer-charged particles has recently lead me to a prejudice that the subcolor symmetry may be most likely not $SU(2)_{sc}$

or $SU(3)_{sc}$ but larger ones such as $SU(4)_{sc}$ allowing only the integer-charged particles. Note that there are many other reasons for preferring $SU(4)_{sc}$ [47, 48, 49], which has been emphasized by myself in the principles of “space-color correspondence” [5], “four-fold way” [47], and “transmuted gauge symmetry” [50].

In any case, it seems very natural to expect that there must exist many exotic particles corresponding to “baryon-like” composite states of subquarks probably with their masses much larger than those of “meson-like” composite states of a subquark and an antishquark. Some of such exotics may behave really exotic. Take for example a “baryon-like” composite state of $(C_0C_1C_2C_3)$, which is not only subcolor- $SU(4)_{sc}$ singlet but also color- $SU(3)_c$ singlet. It is not only electromagnetically neutral but also weakly neutral. However, it strongly interacts with any hadrons due to the van der Waals force induced by the color-singlet state of $(C_1C_2C_3)$ as baryons, the color-singlet states of $(q_1q_2q_3)$. Its mass must be very large (of order of, say, 1 TeV) but it can be absolutely stable. This extremely exotic particle (which we may call “color-ball” or “primitive-hydrogen”) may be another candidate for the missing mass in the Universe. Furthermore, “color-balled nuclei”, the nuclei containing a color-ball, may be the third explanation for the anomalous cosmic ray events recently found by Saito *et al.* [51] after “super-hypernuclei” (or “strangelets”) [52] and “technibaryonic nuclei” or “technibaryon-nucleus atoms” [53].

In the remaining part of this Section, let me present my latest work for predicting the leptoquark and B-L gluon masses in a unified composite model of quark-lepton color interactions [54]. The model is simple as follows:

The most fundamental Lagrangian of subchromodynamics for (massless) subquarks is given by

$$L_{sc} = -\frac{1}{4}(F_{\mu\nu}^A)^2 + \bar{w}_i\gamma^\mu(i\partial_\mu + F\frac{\Lambda^A}{2}A_\mu^A)w_i + |(\partial_\mu - iF\frac{\Lambda^A}{2}A_\mu^A)C_\alpha|^2, \quad (52)$$

where F is the coupling constant, $\Lambda^A (A = 1, 2, 3, \dots, 15)$ is the $SU(4)_{sc}$ matrix, A_μ^A is the subgluon field, and $F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + Ff_{ABC}A_\mu^B A_\nu^C$ with f_{ABC} for the structure constants of $SU(4)_{sc}$. Obviously, this Lagrangian is symmetric not only in the chiral $U(2)_L \times U(2)_R$ transformation for w_{iL} and $w_{iR} (i = 1, 2)$ but also in the $U(4)_c$ transformation for $C_\alpha (\alpha = 1, 2, 3, 0)$. After the gauge-induction [4] of local $SU(4)_c$ from global one or the gauge-transmutation [55] of $SU(4)_{sc}$ into $SU(4)_c$, the effective Lagrangian of quark-lepton color interactions for chroms should become

$$L_c = -\frac{1}{4}(G_{\mu\nu}^A)^2 + |(\partial_\mu - if\frac{\Lambda^A}{2}G_\mu^A)C|^2 + \dots, \quad (53)$$

where f is the effective coupling constant, $G_\mu^A (A = 1, 2, 3, \dots, 15)$ is the 15-plet gluon field, and $G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + ff_{ABC}G_\mu^B G_\nu^C$.

Suppose that a subcolor singlet pair of C_0 and \bar{C}_0 condensates in the vacuum of QSCD, i.e. $\langle \bar{C}_0 C_0 \rangle_0 = (V/\sqrt{2})^2 \neq 0$ where V is a non-vanishing constant. Then, the $SU(4)_c$ symmetry is spontaneously broken down to $SU(3)_c$. Since the effective Lagrangian provides non-vanishing mass terms as

$$L_c = \dots + \left(\frac{fV}{2}\right)^2 \left[\left| \frac{G_\mu^9 + iG_\mu^{10}}{\sqrt{2}} \right|^2 + \left| \frac{G_\mu^{11} + iG_\mu^{12}}{\sqrt{2}} \right|^2 + \left| \frac{G_\mu^{13} + iG_\mu^{14}}{\sqrt{2}} \right|^2 + \frac{3}{4}(G_\mu^{15})^2 \right], \quad (54)$$

a color-triplet of leptoquarks (and their antiparticles)

$$X_1 = (G^9 + iG^{10})/\sqrt{2}, \quad X_2 = (G^{11} + iG^{12})/\sqrt{2}, \quad X_3 = (G^{13} + iG^{14})/\sqrt{2} \quad (55)$$

and the physical B-L gluon $G^B (= G^{15})$ would become massive with the masses

$$m_X = fV/2 \text{ and } m_{G^B} = \sqrt{3/2}(fV/2) (= \sqrt{3/2}m_X) \quad (56)$$

while the ordinary gluons $G^a (a = 1, 2, 3, \dots, 8)$ remain massless.

Note that the physical B-L gluon must be $\sqrt{3/2}$ times heavier than the leptoquarks. Since the presently available lower bound on the leptoquark gauge boson is $m_X \geq 225$ GeV (95% CL)[56], the mass of the physical B-L gluon in this model can be bounded by $m_{G^B} = \sqrt{3/2}m_X \geq 276$ GeV. Since the gluon coupling constant f is determined by $f \equiv (4\pi\alpha_s)^{1/2} \cong 1.2$ (for the renormalization point at $Q^2 = m_Z^2$ [27]), the constant V can be bounded as $V = 2m_X/f \gtrsim 0.4$ TeV. This lower bound on V seems to be consistent with the expectation that the energy scale of QSCD, Λ_{sc} , must be much higher, say $\Lambda_{sc} \gtrsim 1$ TeV, than that of QCD, Λ_c ($\cong 0.2$ GeV [27]). In short, the physical B-L gluon should appear as a neutral, color-singlet, but strongly interacting vector boson which is $\sqrt{3/2}$ times heavier than the leptoquarks and which couples universally with the B-L current of quarks and leptons.

IX. Neutrino Masses and Mixings

One of the latest and hottest news in particle physics is that the Super-Kamiokande Collaboration [57] has found the ratio of up-going to down-going atmospheric muon neutrinos much less than unity and that they have claimed it as an evidence for the non-vanishing mass for the muon and/or tau neutrinos in the analysis based on the neutrino oscillation [58] due to the neutrino mixing among three generations of neutrinos (ν_e, ν_μ, ν_τ). It seems too early to take their claim at its face value before they see more clearly the zenith angle dependence of atmospheric neutrinos or before it is confirmed by the future long-base-line neutrino oscillation experiments by the neutrino beams from KEK, Fermilab, and CERN. However, if they are right, it may be taken as one of the most important discoveries in particle physics since it would indicate not only the non-vanishing mass of neutrinos (which has been searched for for long mostly in the β -decays but in vain so far) but also the breakdown of lepton number conservation [59] (which has been searched for for long mostly in the decays such as $\mu \rightarrow e\gamma$). Note that neither the non-vanishing mass of neutrinos nor the non-zero mixing of neutrinos would indicate by itself anything beyond the standard model for electroweak interactions [1] since both of them can be perfectly accommodated in the standard model. However, one may feel rather uneasy in accepting the Super-Kamiokande report [57], which says, ‘‘The data are consistent with two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with $\sin^2 2\theta > 0.82$ and $5 \times 10^{-4} \text{ eV}^2 < \Delta m^2 < 6 \times 10^{-3} \text{ eV}^2$ at 90% confidence level.’’ In this Section, I shall present a simple model of neutrino masses and mixings whose predictions are consistent not only with such a large mixing and such a small mass-squared difference between ν_μ and ν_τ suggested by the Super-Kamiokande data but also with a small mixing and a large mass-squared difference between ν_e and ν_μ suggested by the LSND data [60] (but not with the solar neutrino deficit [61]).

The non-vanishing small masses for neutrinos have no contradiction with the standard model since fermion masses are all free parameters proportional to Yukawa coupling constants for interactions between the Higgs scalar and fermions in the model. In other words, the possible extreme smallness of the ratio (of the order of 10^{-6}) of a neutrino mass (m_ν of the order of, say, 1 eV) to the electron mass ($m_e \cong 0.5$ MeV) is no more natural in the standard model than the smallness of that of the electron mass to the top quark mass ($m_t \cong 180$ GeV). After all, the standard model would not tell us anything about fermion masses in the tree approximation. Historically, many attempts have been made to explain the small mass ratios of fermions such as $m_e/m_\mu (\cong 1/200)$ by taking a smaller mass as a radiative self-mass (caused by a larger mass) which is finite and calculable in the standard model [62]. Although it may be possible to derive such a small mass ratio as one of the order of 10^{-6} from this picture of radiative corrections (even in the second order), we would not try it here as it seems difficult. On the other hand, the popular see-saw mechanism for producing the non-vanishing small Majorana masses for neutrinos [63] in grand unified theories is easy, provided that neutrinos are not Dirac particles but Majorana ones. However, it suggests the mass ratios of $m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_e^2 : m_\mu^2 : m_\tau^2$, which do not seem to explain the LSND and Super-Kamiokande data. Also, the “see-saw-like mechanism” [64] in supersymmetric grand unified theories suggests the mass ratios of $m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u : m_c : m_t$, which do not seem to explain those data either. In composite models of quarks and leptons [3, 4, 5, 6, 7, 8], not only the smallness of neutrino masses but also that of quark and charged-lepton masses compared to the compositeness energy-scale (of the order of, say, 1 TeV) has tempted us to assume that quarks and leptons (at least of the first and second generations) are taken as almost Nambu-Goldstone (N-G) fermions due to spontaneous breakdown of approximate supersymmetry [21]. In the unified supersymmetric composite model, we have derived the square-root mass sum rules of $m_{\nu_e}^{1/2} - m_e^{1/2} = m_u^{1/2} - m_d^{1/2}$ and $m_e^{1/2} - m_\mu^{1/2} = m_d^{1/2} - m_s^{1/2}$ [37, 38], both of which are very well satisfied with the experimental data. Furthermore, by assuming that quarks and leptons of the first, second, and third generations are almost N-G, quasi N-G [34, 35, 36], and ordinary composite fermions, respectively, we have derived the simple mass relations of $m_\tau = (m_\mu^3/m_e)^{1/2}$ and $m_t = (m_d m_c^3 m_b^2 / m_u m_s^3)^{1/2}$ [39, 40], both of which are well satisfied with the experimental data. However, we have not yet succeeded in deriving any relations among neutrino masses.

Not only the CKM quark-mixing matrix elements (V_{ij} for $i = u, c, t$ and $j = d, s, b$) [23] but also the possible lepton-mixing matrix elements (U_{ij} for $i = \nu_e, \nu_\mu, \nu_\tau$ and $j = e, \mu, \tau$) [65] are all free parameters to be determined by Yukawa coupling constants for interactions between the Higgs scalar and fermions in the standard model. In other words, the possible almost maximal mixing between ν_μ and ν_τ ($\sin 2\theta_{\mu\tau} \cong 1$) is no more natural than the small mixing between d and s ($\sin \theta_C \cong 0.2$). After all, the standard model would not tell us anything about quark and lepton mixings in the tree approximation. Historically, many attempts have been made to explain the small Cabibbo mixing (and especially the “folklore relation” of $\sin \theta_C \cong (m_d/m_s)^{1/2}$ [66]) based on rather arbitrary assumptions. Neither grand unified theories nor supersymmetric grand unified theories would not help us in explaining or predicting the quark and lepton mixing matrix elements. In composite models of quarks and leptons [3, 4, 5, 6, 7, 8], the quark and lepton mixings are naturally taken as mixings between dynamically different composite states of the same

subquarks in different generations [29, 30, 31]. Not only the unitarity of the quark and lepton mixing matrices ($VV^+ = V^+V = 1$ and $UU^+ = U^+U = 1$) has been demonstrated by using the algebra of subquark currents [5] but also the possible momentum transfer dependence of the mixing matrix elements has been predicted. Furthermore, we have derived many relations such as $V_{us} = -V_{cd}^*$, $V_{cb} = -V_{ts}^*$, $|V_{cb}| (= |V_{ts}|) \cong (m_s/m_b)|V_{us}|$, $|V_{ub}| \cong (m_s/m_c)|V_{us}V_{cb}|$, and $|V_{td}| \cong |V_{us}V_{cb}|$, all of which agree well with the experimental data, and have succeeded in determining all the CKM matrix elements by a single parameter (say, the Cabbibo element). However, we have not yet succeeded in predicting any lepton mixing matrix elements but may only suppose that the larger the mass differences between leptons, the smaller the mixings as in the case of quark mixings. If this is the case, it would contradict with the Super-Kamiokande data indicating the small mass-squared difference and the almost maximal mixing between ν_μ and ν_τ ! Thus, we are forced to find a new mechanism for the non-vanishing small neutrino masses and the almost maximal mixing for at least between two generations of neutrinos.

The extremely small mass difference and almost maximal mixing between two neutral particles reminds us of those between K^0 and \bar{K}^0 , which was first pointed out by Gell-Mann and Pais in 1955 [67]. They have asserted that K^0 and \bar{K}^0 , which are eigen-states of strangeness when produced in strong interaction reactions conserving strangeness, should be transformed into either $K_1^0 = (K^0 + \bar{K}^0)/\sqrt{2}$ or $K_2^0 = (K^0 - \bar{K}^0)/\sqrt{2}$, both of which are eigen-states of CP, before disappearing in weak decays conserving CP quantum numbers to a good accuracy. In fact, it was one of the strongest motivations for Pontecorvo to have considered the possibility of neutrino oscillation in 1957 [58]. It is now well-known that the transition of $K^0 \leftrightarrow \bar{K}^0$ occurs due to double exchange of W^+ and W^- between the $(d\bar{s})$ and $(\bar{d}s)$ states and generates an extremely small mass difference of $m_{K_L} - m_{K_S}$ ($\cong 3 \times 10^{-6}$ eV) between K_S ($\cong K_1^0$) and K_L ($\cong K_2^0$) mass eigen-states.

In analogy to this picture of $K^0 - \bar{K}^0$ mixing, suppose that the three neutrinos (ν_e, ν_μ, ν_τ) are originally massless and of no mixings but that they have transitions of $\nu_e \leftrightarrow \nu_\mu$, $\nu_e \leftrightarrow \nu_\tau$, and $\nu_\mu \leftrightarrow \nu_\tau$ due to some unknown mechanism. Then, the neutrino mass matrix has the form of

$$\mathcal{M} = \begin{pmatrix} 0 & \mu & m \\ \mu & 0 & M \\ m & M & 0 \end{pmatrix}, \quad (57)$$

where $m, \mu,$ and M are unknown parameters for the transition matrix elements. Let us further assume that $m \ll \mu \ll M$ for some unknown reason. Then, the mass matrix can be diagonalized approximately by the neutrino-mixing matrix of

$$U = \begin{pmatrix} \cos \theta_{e\mu} & \sin \theta_{e\mu} & 0 \\ -\sin \theta_{e\mu} & \cos \theta_{e\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{\mu\tau} & \sin \theta_{\mu\tau} \\ 0 & -\sin \theta_{\mu\tau} & \cos \theta_{\mu\tau} \end{pmatrix} \begin{pmatrix} \cos \theta_{e\tau} & 0 & \sin \theta_{e\tau} \\ 0 & 1 & 0 \\ -\sin \theta_{e\tau} & 0 & \cos \theta_{e\tau} \end{pmatrix}$$

$$\text{for } \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad (58)$$

into

$$UMU^{-1} \cong \text{diag} \left(-\frac{2m\mu}{M}, M, -M \right) \quad (59)$$

for

$$\tan 2\theta_{e\mu} \cong -2\sqrt{2}m/M, \tan \theta_{e\tau} \cong -\mu/M, \text{ and } \tan \theta_{\mu\tau} \cong -1. \quad (60)$$

Therefore, this simple model predicts many relations such as

$$m_{\nu_1} \cong \Delta m_{\mu\tau}^2 / 2(\Delta m_{e\mu}^2)^{1/2} \cong |m_{\nu_2} - m_{\nu_3}|, \quad (61)$$

and

$$\tan 2\theta_{e\mu} \tan \theta_{e\tau} \cong \Delta m_{\mu\tau}^2 / \sqrt{2}\Delta m_{e\mu}^2, \quad (62)$$

since $\Delta m_{e\mu}^2 \equiv |m_{\nu_1}^2 - m_{\nu_2}^2| \cong M^2$, $\Delta m_{e\tau}^2 \equiv |m_{\nu_1}^2 - m_{\nu_3}^2| \cong M^2$, and $\Delta m_{\mu\tau}^2 \equiv |m_{\nu_2}^2 - m_{\nu_3}^2| \cong 4m\mu$.

Furthermore, from the Super-Kamiokande data [57] of $5 \times 10^{-4} \text{ eV}^2 < \Delta m_{\mu\tau}^2 < 6 \times 10^{-3} \text{ eV}^2$ (at 90% confidence level) and the LSND, BNL E-776, and Bugey Reactor data [60] of $0.3 \text{ eV}^2 < \Delta m_{e\mu}^2 < 2.2 \text{ eV}^2$ and $2 \times 10^{-3} < \sin^2 2\theta_{e\mu} < 4 \times 10^{-2}$ (at 90% confidence level), these relations predict such constraints as $1.7 \times 10^{-4} \text{ eV} < m_{\nu_1} < 5.5 \times 10^{-3} \text{ eV}$, $0.55 \text{ eV} < m_{\nu_2, \nu_3} < 1.5 \text{ eV}$, and $3.6 \times 10^{-3} < \tan \theta_{e\tau} < 7.1 \times 10^{-2}$.

The model has turned out to produce a combination of the ‘‘maximal mixing mechanism’’ for the $\nu_\mu - \nu_\tau$ sector and the ‘‘mini-see-saw mechanism’’ for the $\nu_e - \nu_\mu$ sector. What is left is to explain why the neutrino mass matrix must have the structure indicated in Eq. (57) and why $m \ll \mu \ll M$ and $0.55 \text{ eV} < M < 1.5 \text{ eV}$. The original masslessness of neutrinos can not be explained in the standard model, in grand unified theories, or even in supersymmetric grand unified theories, but can be attributed to the degeneracy of the spinor and scalar subquarks of which neutrinos consist in supersymmetric composite models [6, 7, 8, 21]. Also, the almost vanishing transition matrix element between ν_e and ν_τ can be attributed to conservation of some quantum number which forbids transitions (to the first order) between states with a two generation difference as assumed in deriving such relations as $|V_{ub}| \cong (m_s/m_c)|V_{us}V_{cb}|$ and $|V_{td}| \cong |V_{us}V_{cb}|$ for quark-mixings [29, 30, 31]. However, it seems difficult to explain the smallness of the parameters of m , μ , and M and their ratios of m/M and μ/M at this stage of particle theories as it is to explain that of the quark and charged-lepton masses of lower generations such as m_e , m_μ , m_d , etc. and their ratios such as m_e/m_μ , m_d/m_s , etc.. Again, subquark models of quark and leptons [3, 4, 5, 6, 7, 8] provide us at least a theoretical ground, on which we can imagine that they are small since they are transitions between dynamically different composite states. How to explain the smallness more quantitatively in composite models is a subject for future investigations.

To sum up, I have proposed the simple model of neutrino masses and mixings characterized by the neutrino mass matrix in Eq. (57). It predicts not only the large mixing and small mass-squared difference between ν_μ and ν_τ suggested by the Super-Kamiokande data but also the small mixing and large mass-squared difference between ν_e and ν_μ suggested by the LSND data. Many relations such as in Eqs. (61) and (62) are obtained and

they predict such constraints as $1.7 \times 10^{-4} \text{ eV} < m_{\nu_1} < 5.5 \times 10^{-3} \text{ eV}$, $0.55 \text{ eV} < m_{\nu_2, \nu_3} < 1.5 \text{ eV}$, and $3.6 \times 10^{-3} < \tan \theta_{e\tau} < 7.1 \times 10^{-2}$ from these experimental data. I hope these predictions will be checked by the future cosmic ray, accelerator, and long-base-line neutrino oscillation experiments.

After writing the original form of this Section, I have heard that the neutrino mass matrix in this model is similar to the one suggested by Zee in his specific Higgs model of neutrino Majorana masses [68]. Note, however, that the former is less model-dependent than the latter and that it has produced more general predictions.

X. Conclusion, Further Discussions, and Future Prospects

In this talk, I have proposed the unified composite model of all “elementary” particles and presented many predictions for the weak-mixing angle, the quark-mixing matrix elements, and the subquark, quark and lepton, baryon, weak-boson, and Higgs scalar masses. Furthermore, I have suggested not only the existence of “baryon generations” similar to quark-lepton generations but also that of many exotic particles such as the excited fundamental fermions (quarks and leptons), excited fundamental bosons (gluons, photon, weak bosons, Higgs scalars), leptogluon, barygluon, leptoquarks, and “color-ball”. Some of these predictions have already been checked experimentally and the others will be tested in the near future.

What is left for future theoretical investigations is to try to complete the ambitious program for explaining all the quark and lepton masses by deriving more sum rules and/or relations among them and by solving a complete set of the sum rules and relations. To this end, my private concern is to see whether one can take the remarkable agreement between my prediction of $m_t = (m_d m_c^3 m_b^2 / m_u m_s^3)^{1/2} \cong 180 \text{ GeV}$ and the experimental data as an evidence for the unified supersymmetric composite model. Very lately, I have been more puzzled by the “new Nambu’s empirical quark-mass formula” of $M = 2^n M_0$ with his assignment of $n = 0, 1, 5, 8, 10, 15$ for u, d, s, c, b, t [69], which makes my relation of $m_u m_s^3 m_t^2 \cong m_d m_c^3 m_b^2$ exactly hold. Even more lately, I have been even more puzzled by the relations of $m_u m_b \approx m_s^2$ and $m_d m_t \approx m_c^2$ suggested by Davidson, Schwartz, and Wali (D-S-W) [70], which can coexist with my relation and which are exactly satisfied by the Nambu’s assignment. If we add the D-S-W relations to a set of Eqs. (28) and (42)-(45) and if we solve a set of these seven equations by taking the experimental values of $m_e \cong 0.511 \text{ MeV}$, $m_\mu \cong 105.7 \text{ MeV}$, and $m_W = 80.41 \pm 0.10 \text{ GeV}$ [27] as inputs, we can find the quark and lepton mass matrix of

$$\begin{pmatrix} m_e & m_\mu & m_\tau \\ m_u & m_c & m_t \\ m_d & m_s & m_b \end{pmatrix} \cong \begin{pmatrix} 0.511 \text{ MeV} & 105.7 \text{ MeV} & 1520 \text{ MeV} \\ \text{(input)} & \text{(input)} & \text{(1777.05}^{+0.29}_{-0.26} \text{ MeV)} \\ 3.8 \text{ MeV} & 970 \text{ MeV} & 131.3 \pm 0.2 \text{ GeV} \\ \text{(4.5} \pm 1.4 \text{ MeV)} & \text{(1350} \pm 50 \text{ MeV)} & \text{(173.8} \pm 5.2 \text{ GeV)} \\ 7.2 \text{ MeV} & 150 \text{ MeV} & 5.9 \text{ GeV} \\ \text{(8.0} \pm 1.9 \text{ MeV)} & \text{(155} \pm 50 \text{ MeV)} & \text{(5.3} \pm 0.1 \text{ GeV)} \end{pmatrix} \quad (63)$$

where an agreement between the calculated values and the experimental data or the phenomenological estimates looks reasonable. This result may be taken as one of the most elaborated “modern developments in elementary particle physics”!

The possible substructure of fundamental fermions such as the electron was considered in some detail by McClure-Drell and Kroll [71] and by Low and myself [72] already in the middle of nineteen sixties while that of quarks was pointed out by Wilson and others [73] in the early nineteen seventies. Also, the possible substructure of fundamental bosons such as the weak bosons was discussed in great detail by myself and others [74] in the middle of nineteen seventies. In conclusion, let me repeat what I said in my talks at Paris Conference in 1982 [75] and at Leipzig Conference in 1984 [6]. “It seems to me that it has taken and will take about a quarter century to go through one generation of physics: atomic physics in 1900-1925, nuclear physics in 1925-1950, hadron physics in 1950-1975, quark-lepton physics in 1975-2000, “subquark physics” in 2000-2025, and so on.” “I would like to emphasize that the idea of composite models of quarks and leptons (and also gauge bosons as well as Higgs scalars) which was proposed by us, theorists, in the middle of seventies has just become a subject of experimental relevance in the middle of eighties.” Ironically, a century has past since the discovery of the electron, the “first elementary particle”, and the compositeness of the “elementary particles” would soon be found!

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