Muon Colliders: Features of the Physics Potential

F.F.Tikhonin*
Institute for High Energy Physics, Protvino,
Moscow region, 142284, RF

Introduction

The muon collider concept was introduced many years ago [1].The main impetus for such an idea was craving to get rid of the powerful synchrotron radiation, from which so suffer the cycling electron-positron colliders. The muons being as much as ≈ 200 times heavier than electrons undergo this disease to a much less extent because the intensity of synchrotron radiation is proportional to the $mass^{-4}$. The interest of last several years to the muon collider hugely grown due to several reasons. From the point of view of machine design consideration it was revealed, that ionization cooling concept [2] offers the possibility of making a high luminosity accelerator. Investigations of accelerator physicists of the last several years showed that technical difficulties expected are not insuperable [3]. At the same time the theoretical motivation for muon colliders had been growing with time [4]. From the point of view of physics potential $\mu^+\mu^-$ colliding beams will allow to undertake unique investigations in the yet unexplored (Higgs) sector of the particle physics. because it is well known, that one of the main objectives of present-days elementary particle study are to find Higgs boson and investigate its properties with as high precision as possible [5]. More generally, the task is to study the symmetry breaking mechanism of Electroweak Theory [6]. The question will be investigated over the next several years by the LEP, TEVATRON and LHC.

However there exists the so called intermediate mass region, extended from the LEPII discovery limit ($m_H > 95$ GeV) to $m_H \leq 2m_Z$, which is the most difficult for experimental research. In spite of this difficulty it is hoped that LHC will allow to know the Higgs mass with the precision enough to tune at the resonance with muon collider. The basis for the forementioned tuning is the conception of "Higgs boson factory" analogous to existed "Z^0 boson factories", LEP and SLC. Thereby, the $\mu^+\mu^-$ collider would not merely fill the gap but improve the precision Higgs scalar studying to a greit extent. Therefore such facility might provide a unique vehicle for particle physics research.

More motivations

Higgs boson width in the SM is narrow only in the region of c.m. energy up to $\sqrt{s} \sim 2m_W$, so the Higgs factory is feasible only in this region. To see this lets calculate the cross-section of the process $\mu^+\mu^- \to W^+W^-$ proceeded due to Higgs boson exchange in the s-channel. With the standard notations for the quantities entering the cross-section the result is as follows:

$$\sigma^{\mu^+\mu^- \overrightarrow{H} W^+W^-} = \frac{\pi\alpha^2}{\sin^4\theta_W}\frac{m_\mu^2}{16}\frac{s-4m_\mu^2}{(m_H^2-s)^2+\Gamma_H^2 m_H^2}\Big(\frac{1}{2}\frac{s^2}{m_W^4} - 2\frac{s}{m_W^2} + 6\Big). \quad (1)$$

*E-mail: tikhonin@mx.ihep.su

It is seen from the equation above, that the cross-section due to Higgs-boson exchange reaches its maximum, $\sigma_{max} \cong 0.067pb$ at the c.m. energy $\sqrt{s} \sim 2m_Z$ while the "conventional" cross-section (due to γ, $\nu \, and \, Z^0$ exchange) reaches at this point the value of $\approx$ 15 pb. Partly in view of this it is expedient to search for other processes, where the interaction of Higgs within the lepton sector wold be involved. Two examples of that consideration was presented in the paper [9], where processes $\mu^+\mu^- \to HZ^0$ and $\mu^+\mu^- \to H\gamma$ were proposed as complementary to that of resonant Higgs scalar production and which we will consider in the next two sections. Note, that second of processes above is negligibly small at the tree level in the case of electron-positron collision, but there is hope to observe it at the muon collider.

Associated HZ production in SM

Let's begin with the Bjorken process having muons as the initial state particles, $\mu^+\mu^- \to ZH^0$.

Usually in the course of cross-section calculations one uses the the s-channel diagram alone. So do it we but let's account for masses of initial muons. Then we obtain the following asymptotics of this process at $\sqrt{s} \to \infty$:

$$\sigma^{(s-channel),as}_{\mu^+\mu^- \to ZH^0}|_{m_\mu \neq 0} = \frac{2\pi \, \alpha^2}{\sin^4(2\theta_W)} \cdot g_A^2 \cdot \frac{m_\mu^2}{m_Z^4}. \qquad (2)$$

It is seen, that despite the fact that this diagram is the pure s–channel one, the corresponding cross-section is not falling at high energy, but approaches a constant limit, whose value is equal to $\cong 1.2 \cdot 10^{-2} fb$. Concerning the angular dependence of this cross-section, it could be seen, that this distribution is **flat**, indicating that it comes entirely from the $J = 0$ partial wave. It is obvious, that this behaviour contradicts unitarity condition, which requires $\sigma_{J=0} \leq s^{-1}$ at high energy.

Now we calculate the contribution into cross-section of the cross channels diagrams, t- and u- one. It turns out that corresponding contribution is again equal exactly to the value of $\cong 1.2 \cdot 10^{-2} fb$. Corresponding angular distribution is also flat. At last, let's take into account interference term between two classes diagrams above. We found that it is equal exactly to $\cong -2.4 \cdot 10^{-2} fb$. Adding all the three contributions we obtain result, which removes a seeming contradiction. As it must be, the asymptotic form of cross section for the process under consideration at $\sqrt{s} \to \infty$ acquires "desired" form, e.i. it falls with the energy.

$$\sigma^{as}_{\mu^+\mu^- \to H^0 Z} = \frac{1}{3} \cdot \frac{\pi\alpha^2}{sin^4(2\theta_W)} \cdot \left(g_V^2 + g_A^2\right) \cdot \frac{1}{s}. \qquad (3)$$

The attention must be drawn to the difference between factors containing the coupling constants in (2) and (3). The cancellation obtained reflects the most fundamental property of electroweak theory. This is the consequence of tree-unitarity condition, which must be fulfilled in any nonabelian gauge theory with the symmetry broken in a manner like Higgs one [7, 8].

Meaning to extract information about the Higgs - lepton sector interplay let's look once more at the individual contributions to the cross section. In this respect it is worthwhile to note, that all the three contributions reach their constant asymptotic values not simultaneously. Those, stemming from sum of t-channel and u-channel go to the plateau at the energy around 1 TeV. Negative contribution reaches his minimum value at $\sqrt{s} \cong 2.5 TeV$, while cross-section, corresponding to the s-channel became constant (at finite muon mass) far away from 1–2 Tev region. However, in spite of diffrent characters of contributions behaviour, it seems that there is little hopes of success to distinguish $t-$ and $u-$ channels contribution experimentally owing to small value of μ- meson mass. Because of it in the next paragraph we will consider the process to which the Higgs-Gauge-Boson vertex is not involved.

Associated $H^0\gamma$ production in SM

We now turn to process anologues just considered, but free from the s-channel diagram complication.

Differential cross section of the process $\mu^+\mu^- \to H\gamma$ corresponding to the graphs depicted in Fig.2 for the case when photon is hitting a non-forward detector and neglecting the muon mass apart from that, which is part of muon-higgs boson coupling constant is as follows:

$$\frac{d\sigma^{\gamma H}}{d(\cos\theta)} = \frac{\pi\alpha^2}{2\sin^2\theta_W}\frac{m_\mu^2}{M_W^2}\frac{s^2+M_H^4}{s^2(s-M_H^2)}\frac{1}{(1-\cos^2\theta)}\ , \quad \cos\theta \le 1. \tag{4}$$

Corresponding expression without muon mass neglecting and for the 4π geometry detector see in the paper [9]. Integrated on the angle variable cross section has extremly simple form even without muon mass neglecting, so it is expedient to write out cross section for this case. Introducing in addition to the usual $\beta = \sqrt{1-4m_\mu^2/s}$ the notation $\beta_H = \sqrt{1-4m_\mu^2/m_H^2}$ and integrating the differential cross section, in which muon mass is retained, over $cos\theta$ in the $[-1,1]$ limits, we obtain the cross–section in the following final form:

$$\sigma^{\gamma H} = \frac{\pi\alpha^2}{2\sin^2\theta_W}\frac{m_\mu^2}{M_W^2}\frac{1}{s^2}\frac{1}{\beta}\frac{1}{s-m_H^2}\left[-2m_H^2 s\beta_H^2 + (s^2\beta^4 + m_H^4\beta_H^2)\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right]. \tag{5}$$

The cross section for the case $\sqrt{s} \gg m_\mu$ can be obtained from the expression above by letting $\beta, \beta_H \to 1$ and $\ln[(1+\beta)/(1-\beta)] \to \ln(s/m_\mu^2)$. Evidently, resulting expression will be not much more simpler that of Eq.4.

With the yearly integrated luminosity of $\mathcal{L} \cong 10^3$ fb^{-1} expected at future $\mu^+\mu^-$ colliders, one could collect 20 to 30 $H^0\gamma$ events (detector efficiency is supposed equal 1, and acceptance – 4π). The signal, which mainly consists of a photon and $b\bar{b}$ pairs in the low Higgs mass range or WW/ZZ pairs for Higgs masses larger than $\cong$ 200 GeV, is extremely clean. The background should be very small since the photon must be very energetic and the $b\bar{b}$ or WW/ZZ pairs should peak at an invariant mass M_H. Therefore, despite the low rates, a clean signal gives a good possibility to detect these events.

Discussion and conclusion

Expressions (4) – (5) obtained for the cross-section of the process $\mu^+\mu^- \to H^0\gamma$ are applicable, on the equal foot, to the case of any other scalar particles production.

Note, that at high energy, when initial state masses can be safely neglected, formulae above can be used and for the case of the pseudoscalar production. However, when masses should be taken into account there is sharp diffrence between two cases. This difference is the another reflection of the fine tuning phenomenon, discussed above in the case of reaction $\mu^+\mu^- \to Z^0 H^0$. First and foremost last remark pertains to the axion search problem. Fruitless efforts to find this particle undertaken up to now, produced a widely accepted opinion, that this pseudoscalar is extremely light and weakly interacting ("invisible axion"). However in a recent paper [10] the solution of strong CP – violation problem in QCD has been proposed, which may lead to a heavy axion, $M_a \leq 1$ TeV. Its interaction with usual matter is induced by mixing with axial Higgs boson. For example, in the case of fermions it has the form $\mathcal{L}_{\mathbf{int}} \sim \mathbf{const} \cdot \mathbf{m_f} \cdot (\mathbf{a\tilde{f}i\gamma_5 f})$. A mixing parameters are model dependent but might not be negligible small, therefore this interaction might lead to an observable effects.

Turning back to the Higgs boson we note that apart from the tree-level amplitude for associated γH production considered here there exists one-loop amplitude with heavy particles in loops. This competitive mechanism, equally applicable both to the $\mu^+\mu^-$ and to the e^+e^- colliding beams, was considered in several papers [11],[12], [13], [14], [15] including recently published one [16],[17]. Both mechanisms give the cross-sections which are of comparable size, but there's difference in the c.m. energy behaviours between tree level and one-loop amplitudes. As is seen from the Eqs.5-6 above the tree -level cross section grows when $\sqrt{s} \to m_H$ due to kinematical factor $1/(s - m_H^2)$ in front of it. Contrary to this case the one-loop cross-section is negligible at the threshold and rise with the energy. Comparative pictures of the two types cross-section behaviours are depicted in Fig. 3 of papers [16] and [17] at some representative Higgs boson mass values.

Remarkable feature of those figures is equality of tree level and one loop cross-sections at the practically invariable point $m_H \cong \sqrt{s}/2$, after which the tree level cross section falls rapidly and process dominated by one-loop amplitudes, while up to this point the main contribution cross section receives from the tree level graphs of. At first sight it seems that this difference provides a good opportunity for study the Higgs and lepton sector interrelation. However, we must to realize, that the tree level cross section have "bad behaviour" in the vicinity of point, where $\sqrt{s} \simeq m_H$, so it is need to take care of this region in order to smooth the front edge of cross section curve. Potential cure for this problem is taking into account radiative corrections to the process under consideration. It is hoped to turn to this problem in near future.

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