Gravitational Microlensing

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Foundations of standard theory of microlensing are described, namely we consider microlensing of stars in Galactic buldge, Magellanic Clouds or other nearby galaxies. We suppose that gravitational microlenses lie between an Earth observer and these stars. In contrast to review of Gurevich et al. [13] we mainly consider microlensing by compact objects. Criteria of an identification of microlensing events are discussed. Also we consider also such microlensing events which do not satisfy these criteria (non-symmetrical light curves, chromatic effects, polarization effects). We describe results of MACHO collaboration observations toward Large Magellanic Cloud (LMC) and Galactic buldge in detail. Results of EROS observations toward LMC and OGLE observations toward Galactic buldge are also presented. A comparison of the microlensing theory and observations is discussed in full.

1. Standard model

1.1 Equation of motion of light rays

A standard microlens model is based on a simple approximation of a point mass for a gravitational microlens. Therefore, before a discussion of the microlensing effect we consider a simple gravitational lens model and principles of an image formation by a gravitational lens which has a spherical symmetrical density distribution. If we suppose that gravitational field is weak then photons are emitted from infinity and are bended by gravitational mass (gravitational lens) and move away from a gravitational mass neighborhood to infinity.

First we consider a photon motion in the framework of Newtonian gravitational theory. We suppose that a photon is a particle having a mass $m = \frac{h\nu}{c^2}$.

Let us consider a photon motion near a star having a mass M_*. If a photon is emitted by a source S then we denote an impact distance of the photon motion by p . If we use Cartesian coordinate frame Oxy then the equation of motion of a light ray has the following form:

$$m\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GmM_*}{|\boldsymbol{r}|^3}\cdot\boldsymbol{r}. \tag{1}$$

As follows from Eqn (1), a photon mass is shortened, so there is a light bending effect even in the framework of Newtonian theory. This effect was noted down by Newton, but the first derivation for a bending angle of a light ray by gravitating body was published by J. Soldner in the beginning of the nineteenth century [1].

It is well-known from the analysis of the Newtonian equations of motion that a test particle trajectory may be a hyperbola, a parabola or an ellipse.

The quantitative criteria of different types of a particle trajectory consist of a comparison of a potential energy of particle in the gravitational field ($U = \frac{GM_* h\nu}{c^2 p}$ for the case) and a kinetic energy ($E = h\nu$ for the case). Since the test particle is a photon for our case, so the criterion is the fraction of a gravitational potential of a body and a square of speed of light $\frac{GM_*}{c^2 p}$. The fraction is much less unity for considered astronomical models, so the trajectory is a hyperbola and kinetic energy of a photon is much greater than its potential energy.

Below we will analyse a light ray displacement along the axis Oy, which is perpendicular to an original velocity of a photon. Since a light ray moves practically along the axis Ox, in zeroth order in the parameter $\frac{GM_*}{c^2 p}$ we have the following equation of motion $x = ct$. If we express t via x and substitute it in Eqn (1), then we obtain the equation in the parametric form $y(x)$

$$\frac{d^2 y}{dx^2} = -\frac{GM_* y}{c^2 (x^2 + y^2)^{3/2}}. \tag{2}$$

We suppose that the displacement is a very small one, thus we assume $y \approx p$ in the right hand side of the Eqn (2). So it is possible to calculate an integral of the right hand side of the Eqn (2). Really, using the substitution $x = p \mathrm{tg} \phi$, we obtain

$$-p \int_{-\infty}^{x} \frac{dx}{(x^2 + p^2)^{3/2}} = -p^{-1}(\sin\phi + 1). \tag{3}$$

We note that $\frac{dy}{dx}$ is a tangent line for the photon trajectory, the difference of the values $\frac{dy}{dx}$ for $+\infty$ and $-\infty$ is equal to the bending angle of a photon in the gravitational field of a star M_*:

$$\Delta\varphi = \left.\frac{dy}{dx}\right|_{x=-\infty} - \left.\frac{dy}{dx}\right|_{x=+\infty} = \left.\frac{dy}{dx}\right|_{\phi=-\pi/2} - \left.\frac{dy}{dx}\right|_{\phi=+\pi/2} = -\frac{2GM_*}{c^2 p}. \tag{4}$$

We obtain the bending angle which is equal to a half of a correct value of bending angle. The difference is connected with an usage of non-relativistic approximation, but a photon is a relativistic particle moving with the limiting speed (the speed of light).

In the framework of general relativity using a weak gravitational field approximation the correct bending angle is described by the expression which is greater in two times than the right hand of the Eqn (4). Really the bending angle is defined by the following expression:

$$\delta\varphi = -\frac{4GM_*}{c^2 p}. \tag{5}$$

The derivation of the famous Einstein's formulae for the bending angle of light rays in gravitational field of a point mass M_* is practically in all monographs and textbooks on general relativity and gravity theory (see, for example books of Landau & Lifshitz [3], Moller [2] and Zakharov [4]).

In the framework of general relativity the light ray bending effected was predicted by A. Einstein in 1915 and firstly was observed by Sir A. Eddington for bending of rays by the Solar gravitational field near its surface. The angle is equal to $1.75''$, and Einstein prediction was confirmed by observations.

1.2 Point lens equation

Since a photon moves practically along straight lines far from a gravitating body, we approximate the photon trajectory by two straight lines which are intersected near the body D (Fig. 1). The angle between the lines α demonstrates the photon bending in the gravitational field of the body D.

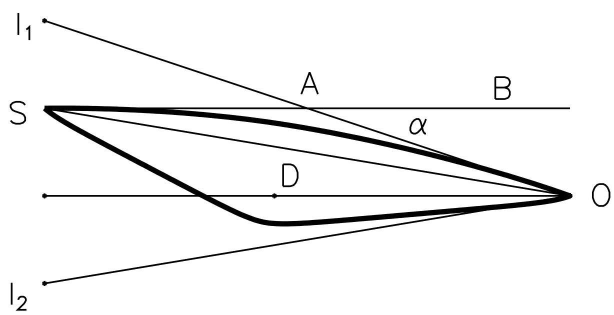

Figure 1: Formation of images and light rays bended by the gravitational field of a body.

Two rays of light, which lie in opposite sides respectively the gravitating body, are deflected to the gravitating body. If a source S lies far away from the body D then the rays begin to converge and intersect in some distant point O (Fig. 1). If we suppose that an observer is in the point O, he will see two images (I_1, I_2) of one source S. Really that is gravitational lens effect. In Fig. 1 three physical bodies are shown, namely a source S, a gravitating body D and an observer O. Trajectories of light rays from S to O are shown by two bold solid lines. We use also the following notations: D_{ds} is the distance from the source S to the lens D; D_d is the distance from the lens D to the observer O; D_s is the distance from the source S to the observer O. We draw plane via the point S and we suppose that the plane is perpendicular to a light ray trajectory. The plane is called as a plane source. Similar we draw the plane via the gravitational lens D. The plane is called as a lens source. We use also the following notations for the angles: θ is an angle between a direction to the gravitating body D and a direction to the source S, θ_1 is an angle between a direction to the gravitating body D and an apparent direction to the source S, I_1 I_2 are images (mirages) of the source.

It can be seen from the figure that we have the following expressions for the angles:

$$\alpha = \beta_1 + \beta, \tag{6}$$

$$\theta_1 = \theta + \beta, \tag{7}$$

$$\beta_1 \cdot D_{ds} = \beta \cdot D_d, \tag{8}$$

where the angle β is expressed in radians, $D_{ds}\,, D_d$ are the distance from the source to the gravitational lens and from the gravitational lens to the observer respectively. From

Eqns $(7-8)$ we obtain the quadratic equation for the angle θ_1 which determines apparent positions of images respective a direction toward a gravitational lens,

$$\theta_1^2 - \theta\theta_1 - \theta_0^2 = 0, \tag{9}$$

where θ is an angle between the direction toward a gravitational lens (GL) and a true position of a distant lensed source, θ_0 is an angular radius of Einstein cone which is defined as

$$\theta_0^2 = \frac{4GM}{c^2} \cdot \frac{D_{ds}}{(D_{ds} + D_d) \cdot D_d}.$$

Eqn (9) is called the gravitational lens equation for the case of spherical symmetric point lens. The equation has two real roots, namely

$$\theta_1 = \frac{1}{2}\theta + \frac{1}{2}\sqrt{\theta^2 + 4\theta_0^2},$$

$$\theta_2 = \frac{1}{2}\theta - \frac{1}{2}\sqrt{\theta^2 + 4\theta_0^2},$$

corresponding to two images of a source S. The angular distance between images is equal to $\sqrt{\theta^2 + 4\theta_0^2}$.

According to previous arguments we wrote about two images. However these two images are formed not always. Really we used the assumption that the sizes of a gravitating body D are infinitesimal and the Eqn (5) is valid for any impact parameter. Actually if the impact parameter is less than the radius R_D of a gravitating body D or

$$R_D > D_d\theta_2$$

than the image I_2 disappears for an observer O (the light ray moving along the trajectory with the impact parameter is absorbed by a matter of a gravitational lens if it is non-transparent). Therefore, only one image of a source is formed for this case. That is the reason why Earth's observer does not see two images during a solar eclipse in spite of an existence of a set of stars which lie near the line drawing via the solar center and the observer (we recall that the angular solar size is about a half of an angular degree, which is much greater than the Einstein's cone size of the Sun since the distance between Earth's observer and the gravitational lens (Sun) is equal to 1 astronomical unit).

For a spherically symmetric gravitational field of a body D an image of circular star S is transformed by the gravitational lens into two "Moon's crescents", which are reflected mirror-likely respectively each other (Fig. 2) ([[5], [6], [7], [8], [9], [4]). Their sizes and brightness are different, but the total shine is greater than the original shine of the (unlensed) source S. The discovery of microlensing effects is based on the property usage.

It is necessary to note that according to the equivalence principle two bodies with different masses fall with the same acceleration in a gravitational field. Therefore, two photons having different frequencies (different energies and thus different masses) are accelerated identically in a gravitational field. In other words, photons lying in different bands are bended identically in a gravitational field of a body D. This property is called the achromatism of the microlensing effect. Possible violations of the property may be connected with complicated structure of a source S, the violations will be discussed below.

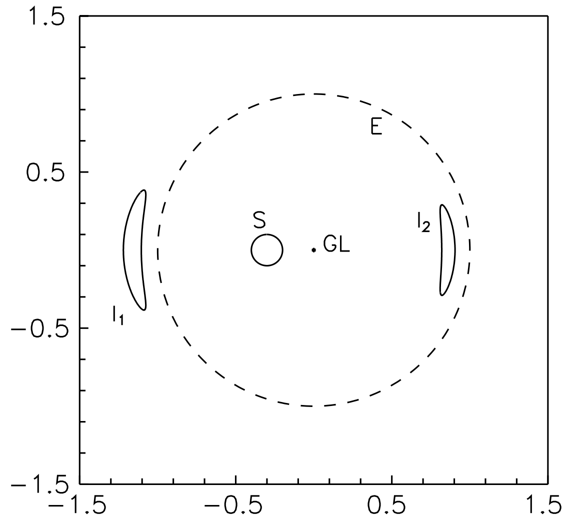

Figure 2: Two images of a circular source, which are formed by the point gravitational lens (for the case when the angular distance between a gravitational lens and the source centre is $d = 0.3$ in the Einstein – Chwolson units).

The gravitational lens effect is a formation of several images instead one. We have two images for a point lens model (Schwarzschild lens model). The angular distance between two images is about angular size of so called Einstein's cone. The angular size of Einstein's cone is proportional to the lens mass divided by the distance between a lens and an observer. Therefore, if we consider a gravitational lens with typical galactic mass and a typical galactic distance between a gravitational lens and an observer then the angular distance between images is about few angular seconds; if we suppose that a gravitational lens has a solar mass and the distance between the lens and an observer is about several kiloparsecs then an angular distance between images is about angular millisecond.

If a separation angle is $\sim 1''$ then one may observe two images in optical band although that is a complex problem, but it is impossible to observe directly two images by Earth's observer in the optical band if a separation angle is $\sim 0.001''$. Therefore, the effect is observed on changing of a luminosity of a source S.

Let us consider a change of a luminosity of images during microlensing. First, we determine a luminosity either of two images.

Let us return to Fig. 1. A light source S, a dark body D and an observer O have a peculiar motion. It is possible to decompose the motion of each of these three bodies on two vectors, one of them is perpendicular to the straight line OD and the second vector is parallel to the line. The parallel component of the velocity changes the basic astrophysical parameters of the model, for example the size of Einstein's cone. However the change is very small and we will neglect its influence. The perpendicular components of the velocity are added together and there is a motion of a source in the gravitational lens plane respectively a dark body D.

Let us consider a source motion across the Einstein's cone. (Fig. 3). The real motion of a light source S is shown by solid straight line, but motions of images are drawn by two dashed lines I_1 and I_2. Einstein – Chwolson circle is shown by the dashed line also. It is possible to find θ which is an angular distance between a light source S and a gravitational lens D from the following expression

$$\theta^2(t) = \Omega^2 t^2 + \theta_p^2.$$

We choose the time $t = 0$ according to the following rule: the time $t = 0$ corresponds to the minimal angular distance (θ_p), between S and D. If we assume that an observer and a lens are fixed then Ω is an angular velocity of a source on the celestial sphere. Thus, the distances between a dark body and the first and the second images are equal to

$$\theta_1(t) = \frac{1}{2}\sqrt{\Omega^2 t^2 + \theta_p^2} + \frac{1}{2}\sqrt{\Omega^2 t^2 + \theta_p^2 + 4\theta_0^2}$$

and

$$\theta_2(t) = \frac{1}{2}\sqrt{\Omega^2 t^2 + \theta_p^2} - \frac{1}{2}\sqrt{\Omega^2 t^2 + \theta_p^2 + 4\theta_0^2}$$

respectively.

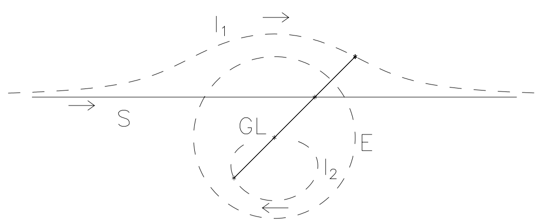

Figure 3: The motion of a source and its images which are formed by point gravitational lens. The straight line motion of a source is considered. The directions of motions of a source and images were shown by arrows.

According to the expressions, we draw trajectories of visible motions of these two images ($I_1(t)$ и $I_2(t)$) on the celestial sphere. The trajectories are shown by dashed lines in Fig. 3. The motion direction of the source S are shown by the arrow. The motion direction of the image I_1 coincides with the source motion direction, but the motion direction of the image I_2 is opposite respectively the source motion direction.

The auxiliary variable $u(t)$ introducing above is determined by the expression

$$u(t) = \sqrt{1 + \frac{4\theta_0^2}{\Omega^2 t^2 + \theta_p^2}},$$

therefore the total luminosity of two images

$$I(t) = I_0(u(t) + u^{-1}(t))/2$$

is a symmetrical function on time respectively the time $t = 0$ (Fig. 4).

If the source S lies on the boundary of the Einstein cone ($\theta(t) = \theta_0$), then we have $A = 1.34$. We note that the total time of crossing of Einstein cone as T_0, so

$$T_0 = 2\frac{\sqrt{\theta_0^2 - \theta_p^2}}{\Omega}.$$

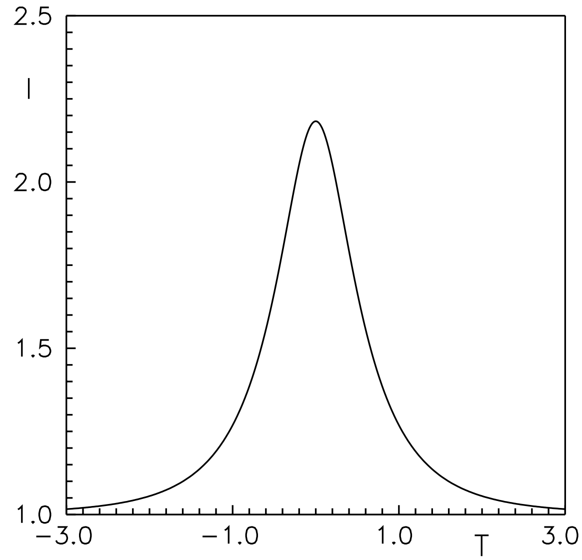

Figure 4: Typical dependence of the amplification factor on time (in units $T_0/2$).

Sometimes the microlensing time is defined as a half of T_0. If we suppose that $D_d < D_{ds}$, then

$$T_0 = 3.5\ months \cdot \sqrt{\frac{M}{M_\odot}\frac{D_d}{10\,kpc}} \cdot \frac{300\,km/s}{v},$$

where v is the perpendicular component of a velocity of a dark body.

We will give numerical estimations of parameters of the microlensing effect. If the distance between a dark body and the Sun is equal to $\sim$ 10 kpc, then the angular size of Einstein cone of the dark body having approximately solar mass is equal to $\sim 0.001''$ or the linear size of Einstein cone is equal to about 10 astronomical units. If we suppose that the perpendicular component of a velocity of a dark body is equal to $\sim$ 300 km/s (that is a typical stellar velocity in Galaxy), then a typical time of crossing Einstein cone is about 3.5 months. The luminosity of the source S is changed with the time.

For observations of extragalactic gravitational lens the typical time for changes of light curve is very long $\sim 10^5$ years for its direct observations. Therefore, extragalactic gravitational lenses are discovered and observed by resolving different optical components (images) since typical angular distances between images are about some angular seconds because of a great mass of a gravitational lens. If a gravitational lens is a galaxy cluster then the angular distances between images may be even about some minutes. For the identification of gravitational lenses, observers compare typical features and spectra of different images. It is impossible to resolve different components during microlensing but it is possible to get and analyse a light curve in different spectral bands.

1.3 Non-compact microlenses

The character of gravitational microlenses is unknown till now although the most widespread hypothesis assumes that they are compact dark bodies such as brown (or cold white) dwarfs. Nevertheless they could be another objects, in particular, dark objects consisting of sypersymmetrical weakly interacting particles (neutralino) as discussed in the papers of [10, 11, 13, 16, 15, 14, 17, 18, 19]. The authors showed that the neutralino stars may be formed in the early stages of the evolution of the Universe and be stable during cosmological timescales.

Microlensing of a distant star by a neutralino star is considered in the section.

2. Gravitational microlens observations

2.1 Introduction

For the first time a possibility to discover microlensing using observations of star light curves was discussed in the paper of Byalko [20]. Systematical searches of dark matter using typical variations of light curves of separate stars from millions observable stars start after Paczynski's discussion of the halo dark matter discovery using monitoring stars from Large Magellanic Cloud (LMC) [21]. We remark that in the beginning of the nineties new computer and technical possibilities providing the storage and processing of huge massive of observational data were appeared and it promoted at the rapid realization of Paczynski's proposal. Griest offered to call the microlenses as MACHO (Massive Astrophysical Compact Halo Objects) [22]. Besides MACHO is the name of the project of observations of the same name US-English-Australian collaboration which observes the LMC and Galactic bulge using 1.3 m telescope of Mount Stromlo observatory in Australia. Some information about the experiment is in the sites `http://wwwmacho.mcmaster.ca/` and `http://wwwmacho.anu.edu.au/`. Information about alert microlensing events from current observational data of MACHO collaboration in real time is in the site `http://darkstar.astro.washington.edu/`.

First papers about microlensing discovery were published by the MACHO collaboration [23] and French collaboration EROS (Expérience de Recherche d'Objets Sombres) [24]. Some information about EROS experiment is in the sites `http://www.lal.in2p3.fr/EROS/eros.html`.

First papers about the microlensing discovery toward Galactic bulge were published by US-Polish collaboration (Optical Gravitational Lens Experiment), which used 1 m telescope at Las Campanas Observatory. Some information about the OGLE experiment is in the sites `http://www.astrouw.edu.pl` and `http://www.astro.princeton.edu/ ogle/`. The results of the OGLE collaboration which include the photometry of OGLE microlensing event candidates, papers of the OGLE collaboration, as well as regularly updated status report can be found over Internet from the host `sirius.astrouw.edu.pl` (148.81.8.1) using "anonymous ftp" service.

We note that the addresses of sites were changed (or the access modes for the data) in past, probably the addresses will be changed in future. However, we indicated the sites for the collaborations, which the groups used in 1999 (perhaps the addresses will be used in future).

2.2 Microlensing features

The event corresponding to microlensing may be characterized by the following main features, which allow to distinguish the microlensing event and a stellar variability (see for example [25, 4]).

- Since the microlensing events have a very small probability, the events should never repeat for the same star. The stellar variability is connected usually with periodic (or quasi-periodic) events of the fixed star.

- In the framework of a simple model of microlensing when a point source is considered, the microlensing effect must be achromatic (deviations from achromaticity for non-point source were considered, for example in the paper [28]), but the proper change of luminosity star is connected usually with the temperature changes and thus the light curve depends on a colour.

- The light curves of microlensing events are symmetrical, but the light curves of variable stars are usually asymmetric (often they demonstrate the rapid growth before the peak and the slow decrease after the peak of a luminosity).

- Observations of microlensing events are interpreted quite well by the simple theoretical model, but some microlensing events are interpreted by more complicated model in which one can take into account that a source (or a microlens) is a binary system, a source has non-vanishing size, the parallax effect may be.

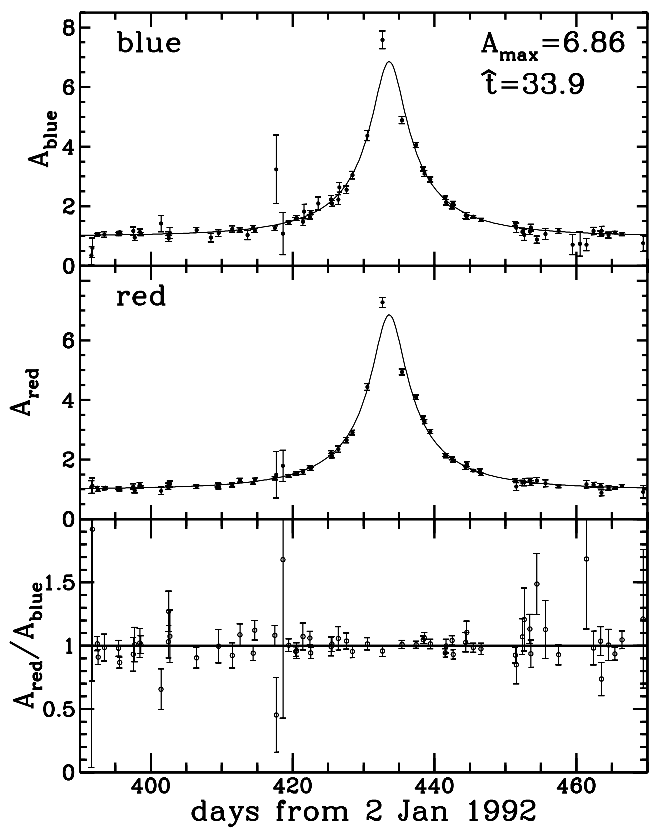

Figure 5: The first microlensing event which was detected by the MACHO collaboration during microlensing searches towards LMC (Alcock et al. [23]).

The typical features of the light curve of the first microlensing event observed by the MACHO collaboration in the LMC are shown in Fig. 5, where the light curves are shown for two spectral bands* . The light curve (in two bands) is fitted by a simple model well enough, but the ratio of luminosities for the bands is shown in the lower panel of figure (the ratio shape is adjusted with the event achromaticity). However, one can note that near the maximal observable luminosity the theoretical curve fits the data of observations not very well.

Now one can carry out accurate testing the achromaticity and moreover the stability of the source spectrum during microlensing event with the Early Warning systems implemented both by MACHO [29] and OGLE [30] collaborations. This allows to study the source properties using large telescopes and to organize intense follow-up studies of light curves using telescope network around the globe [29].

In addition to the typical properties of individual microlensing events, Roulet and Mollerach note that the population of observed events should have the following statistical properties [25]:

Unlike a star variability microlensing events should happen with the same probability for any kind of star therefore the distribution of microlensing events should correspond to the distribution of observed stars in the color-magnitude diagrams** .

The distribution of the maximal amplification factor $A_{\max}$ should correspond to a uniform distribution of the variable $u_{\min}$.

The distributions of the amplification $A_{\max}$ and the microlensing event time T should be uncorrelated.

The basic point of the microlensing discovery is a very small probability of the microlensing event. It is not difficult to obtain a simple estimate of the optical depth for the searches towards LMC, since it was obtained the following estimate $\tau \simeq V^2/c^2$ in the previous section and thus, we obtain $\tau_{\text{halo}} \simeq 5 \times 10^{-7}$ for the halo case [25]. The average duration of microlensing events is connected with the average velocities in the halo as $T \simeq R_E/\sigma \simeq 100 \text{ days}\sqrt{m/M_\odot}$ (using the assumption $x = D_d/D_s = 1/4$). Hence one can obtain that, for lenses with masses in the range $10^{-2} - 10\ M_\odot$, the typical event durations is between a week and an year. Using the estimates one can conclude that several million stars should be monitored for more than a year and it is necessary to measure the observable luminosity of stars with the typical time interval about several days. In the case when the lenses have masses in the range 10^{-7}–$10^{-4} M_\odot$, the events last less than a day, the event rates $\Gamma \propto \tau/\langle T \rangle$ are much larger than the event rates for the lenses of the star mass. To discover the lenses in this mass range it is necessary to observe with much better time coverage, i.e. performing several measurements per night. Thus, for the exploration of the possible range of the lens masses $10^{-7} < m/M_\odot < 10$ it is necessary to perform measurements of the same star fields several times per night and with the time interval about a day.

Roulet and Mollerach present a simple estimate of the optical depth $\tau \simeq 10^{-6}$ for the bulge stars which are lensed by stars in the bulge and the disk with typical event durations of a few weeks, so that in this case also several million stars need to be monitored for more than a year to get reliable statistics [25].

* A more recent MACHO fit to the observed amplification of this event gives $A_{\max} = 7.2$.

** However as Roulet and Mollerach noted that for observations in the bulge since the observed star have non-negligible spread along line of sight and therefore the optical depth is significantly larger for the star lying behind the bulge, thus the lensing probabilities should increase for the fainter stars [25].

Since for the microlens searches one can monitor several million stars during several years, the ongoing searches have focused on two targets: a) stars in the Large and Small Magellanic Clouds (LMC and SMC) which are the nearest galaxies having line of sight which goes out the Galactic plane and well across the halo; b) stars in the Galactic bulge which allow to test the distribution of lenses near to the Galactic plane.

2.3 Searches towards LMC and SMC

Now there some collaborations which are devoted to the searchers of microlensing events toward Magellanic Clouds. Below we will discuss results of MACHO and OGLE collaborations.

EROS experiment

The first experiment is realized by the French collaboration which actually had two different programs at La Silla Observatory in Chile [25]: i) they have used a CCD camera in 40 cm dedicated telescope to make short exposures (10 min each) so as to be able to test short duration events; ii) they analysed plates from 1 m Schmidt telescope which made two exposures per night in different colors from which they have followed the light curves of several million stars since 1991. Using the information in the plates which had been collecting since 1990 one can obtain several million light curves. The plates was obtained using the Schmidt telescope with the 1 m aperture and the 3 m focus length. Each plate has the size 30 cm $\times$ 30 cm and corresponds to the celestial sphere element $5^\circ \times 5^\circ$, so it covers the larger part of the LMC square.

During one night of observations EROS collaboration got usually two plates (the red and blue filters were used), the time exposure for each plate was about 1 hour. During the first winter of observations in 1990 – 1991 EROS collaboration got 28 plate pairs (total 56 plates). The collaboration had about 200 plate pairs to the end 1994. The data from the plates were digitized using MAMA* of Paris Observatory, the digital data density was about 10^4pixels/mm^2. Digitizing one plate took 6 hours (the data size was about 1.6 Gb), each image had 784 cadres, each cadre had 1024 $\times$ 1024 pixels, each pixel corresponded to 0.67 angular square seconds on the celestial sphere. Analysing 1 sm^2 of the plate one can estimate the luminosities of 10000 stars. However, only about 50 % observational stars (about 4×10^6 stars) were suitable for microlensing searches [38].

CCD camera (for searching events with respective small event duration) was fixed in the focus of the 40-cm dedicated telescope in the end of 1991. The camera consists of 16 separate matrices, has about 4×10^6 pixels and may be used for observations of the celestial sphere element with the size $0.4^\circ \times 1^\circ$. One pixel corresponds to the square angle which is equal to 1.1 angular square seconds. Typical time exposure is about 10 min. The collaboration got 46 images in the blue and red colors [31]. The goal of the observations with using the camera is searches of microlensing events which have respective small duration ($\gtrsim 1$ *hour*). Using CCD camera monitoring about 10^5 stars in the bar of LMC was realized.

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* MAMA is the abbreviation of the French words Machine Automatique a Mesurer pour l'Astronomie.

From July 1996 the EROS collaboration started to use 1 m dedicated telescope MARLY for microlensing searches which is located in Observatory of ESO [32] (the observations using the telescope are called EROS 2). This telescope may carry out observations simultaneously in the blue band ($\lambda \in [420\ nm, 720\ nm]$ with maximal sensibility $\lambda \sim 560\ nm$) and in the red band ($\lambda \in [620\ nm, 920\ nm]$ with maximal sensibility $\lambda \sim 760\ nm$) using dichroic cube for splitting the beam of light. CCD-camera is fixed in the each channel, each camera has 8 CCD-matrices with 2048×2048 pixels. The field sizes are 0.7 angular degree (right ascension) $\times$ 1.4 angular degree (inclination). The pixel size is about 0.6 angular seconds. As the main targets for observations the collaboration chose the stars near Galactic Centre in Galactic plane, stars in LMC and SMC.

MACHO collaboration

Australian - US - English MACHO collaboration carries out the observations using 1.27 m telescope of the Mount Stromlo Observatory near Canberra. The collaboration uses the optical corrector with the field about $0.7^\circ \times 0.7^\circ$, the system for the dichroic splitting the beam of light. The system gave a possibility to get images simultaneously in the red and the blue bands. Two large CCD-cameras are fixed in the focuses of the system, each camera has 4 chips containing 2048×2048 pixels. The typical time exposure is about 300 seconds, so during the clear night one can get up to 60 images. The collaboration had got 50000 images by October 1996 [33]. The Galactic bulge was observed when LMC and SMC were too low on the celestial sphere.

The PLANET project

As Albrow et al. noted, from 1995 the program PLANET:= Probing Lensing Anomalies NETwork was started. The aim of the project is to follow the announcements of real time detection of microlensing events (currently implemented by the OGLE and MACHO collaborations) with frequent multi-color observations on four telescopes: 0.6 m telescope of the Perth Observatory at Bickley in Australia, the 1 m telescope near Hobart in Tasmania, 1 m of the South African Astronomical Observatory at Sutherland in South Africa and Dutch - ESO 0.92 m telescope at La Silla in Chile.

Observational results of the EROS collaboration

Observing more than 80000 stars during 10 months (since 21 August 1992 till 31 March 1993) using CCD-camera, the EROS collaboration has no found no events. If we consider the standard halo model ($4 \times 10^{11} M_\odot$), then one can estimate the expected number of microlensing events using Monte Carlo simulations. The theoretical estimate of the expected event number is calculated using the assumption that all microlenses have the same mass. Since the estimated number of microlensing events is greater than 2.3 for the lens mass in the range $5 \cdot 10^{-8} < M_d/M_\odot < 7 \cdot 10^{-4}$, then basing on results of the observations Magneville concluded that with the probability 90 % the lenses could not be formed a component which would contribute essentially in the halo mass [31]. In the case when the lens mass is in the range $3 \cdot 10^{-7} < M_d/M_\odot < 1.5 \cdot 10^{-5}$, the expected number of microlensing events is greater than 6.9, and Magneville gave the statistical conclusion that the total mass of the microlenses with a such mass could not form more than one third of the halo mass [31].

Table 1: The teoretical estimates of microlensing events for the standard halo model in dependence on the microlens masses which could be observed in the EROS experiment using CCD camera (Table from the review of Ansari [38]).

The lens mass $M_\odot$	the event number
10^{-7}	5.6
10^{-6}	9.7
10^{-5}	4.3
10^{-3}	1.9

Using observations of the first three years in the Schmidt telescope with a total exposure $E = 3\ yr \times 3.33 \cdot 10^6 stars$, the EROS collaboration found two candidate of microlensing events with durations $T_1 = 23$ days and $T_2 = 29$ days[24]* .

We recall that efficiency depends on the event duration ($\epsilon = \epsilon(T)$), so if we suggest that the EROS collaboration found one microlensing event then one can obtain the following estimate of the optical depth towards the LMC**

$$\tau_{est}^{EROS} \equiv \frac{\pi}{2E} \sum_{events} \frac{T_i}{\epsilon(T_i)} < 4 \times 10^{-8}. \tag{10}$$

From July 1996 to February 1997 and since July 1997 using the modified equipment the EROS collaboration has been carrying out the observations of the SMC region, which is covered by 10 fields with total square about 10 square degrees and with the maximal number of observable stars [32]. From 1996 to 1997 they got from 60 up to 120 images for each field. The repeated observation for each star field was realized after 2 – 4 days. The total exposure was from 5 up to 15 min. Thus, the EROS collaboration got the light curves of 5.3 million stars. After processing these observations the EROS collaboration reported about one candidate of the microlensing event, or more exactly speaking the authors wrote that "one star had a light curve which could be interpreted best of all as the microlensing event by an invisible body" [32]. The crossing time of the Einstein radius is about 123 days. The EROS collaboration estimates the lens mass as $2.6^{+8.2}_{-2.3}\ M_\odot$, the maximal amplification factor is equal to 2.6. Analysing the parallax effect they shown that if the lens is located in the halo then its mass should be not less than 1.2 $M_\odot$, but if the lens is located in SMC then its mass is about 0.1 $M_\odot$. They estimated the optical depth towards the SMC as $\sim 3.3 \times 10^{-7}$.

Observational results of the MACHO collaboration

As Sutherland et al. reported, by October 1996 the MACHO collaboration had completed the analysis of more than two years observations for 22 well investigated regions

* However, Ansari et al. reported that the event EROS #2 is eclipsing binary system possibly with an accretion disk. The system has the characteristic period about 2.8 days [36]. Paczynski noted that probably the event EROS #1 is an emission line Be type star (or a variable star of a new kind [37]). The new class of variable stars was found as a serendipity result of the MACHO collaboration.

** Assuming that the EROS collaboration found two microlensing events Ansari et al. estimated the optical depth towards the LMC as $\tau_{est}^{EROS} = 8.2 \times 10^{-8}$ [38].

of LMC, which have about 8 million stars for each star from 300 up to 800 luminosity observations are collected [33, 34]. Thus, the total exposure was about $1.8 \times 10^7 stars \cdot yr$. First, the MACHO collaboration selected only the microlensing event candidates which satified strong selection rules, namely the light curve have to characterized by the essential changes of an luminosity (the maximal amplification factor was $A_{\max} > 1.75$), but outside the amplification region the star luminosity must be approximately constant. Then the selection rules were slightly changed since the MACHO collaboration gained the experience in analysis of the observational data got during the observation towards Galactic bulge. In particular, the selection rules on the standard shape and the achromaticity were reduced, but the requirements on the statistical level and the amplification factor were strengthened. Thereby, the events # 2 and # 3, which were observed during the first year of observations and selected firstly, do not correspond to the new selection rules, but the researchers found new candidates which were observed during the first year and correspond to the new selection criteria. During processing observational results, it was found 12 objects satisfying the final criteria, four of them correspond only two stars (or the stars are in the intersection region of two monitored neighboring fields, two objects were excluded from the consideration since, probably, their light curves cannot be explained by microlensing and as Sutherland et al. noted one of the stars is most probably supernovae [33]. As a result it was found 8 candidates of microlensing events with typical duration from 34 up to 145 days. The events are numbered by 1, 4 - 10 (the numbers 2 and 3 are missed since the events, which were selected earlier, do not satisfy the new criteria). Six from the eight candidates correspond good enough to the standard light curve for a simple microlens model; three of them (which have numbers 5, 7 and 9) demonstrate weak dependence on color. The event # 9 has the light curve with two typical maximums which corresponds most probably to the binary lens [39]. The event # 10 has slightly non-symmetrical light curve. Probably a variable star corresponds to the event but Sutherland et al. suggested that there was microlensing binary background star [33]. The final statistical results do not depend on the inclusion (or the exclusion) of this event into consideration. Sutherland et al. think that the selected events are actually connected with microlensing and can not be connected with errors of observations, such as the influence of cosmic rays and other causes. Besides the event MACHO # 1 was also confirmed by the EROS collaboration, but the event # 4 was detected in real time and observed using other telescopes [33]. Sutherland et al. note that it is very difficult to exclude the proper stellar variability however some microlensing candidates demonstrate the essential growth of a luminosity and it is very difficult to explain the light curves by another way (not using the microlensing model)[33]. The spectral data were got also for the event # 4 which confirms the hypothesis of microlensing. As Alcock think that the distribution of the maximal amplification factor distribution and color-magnitude diagram agree with the estimated parameters [40]. In October 1996 MACHO collaboration think that at least five from the suggested event candidates are connected with the microlensing manifestation, however Alcock et al. noted that if only the "high quality" events are considered (for example, the events 1,4,5 and 9), then the probability of the observable distribution for the variable $A_{\max}$ is very small [40].

Allowing to Sutherland et al. [33] we suppose that the halo is formed by objects with the same mass, then the expected number of microlensing events is shown in Fig. 6a.

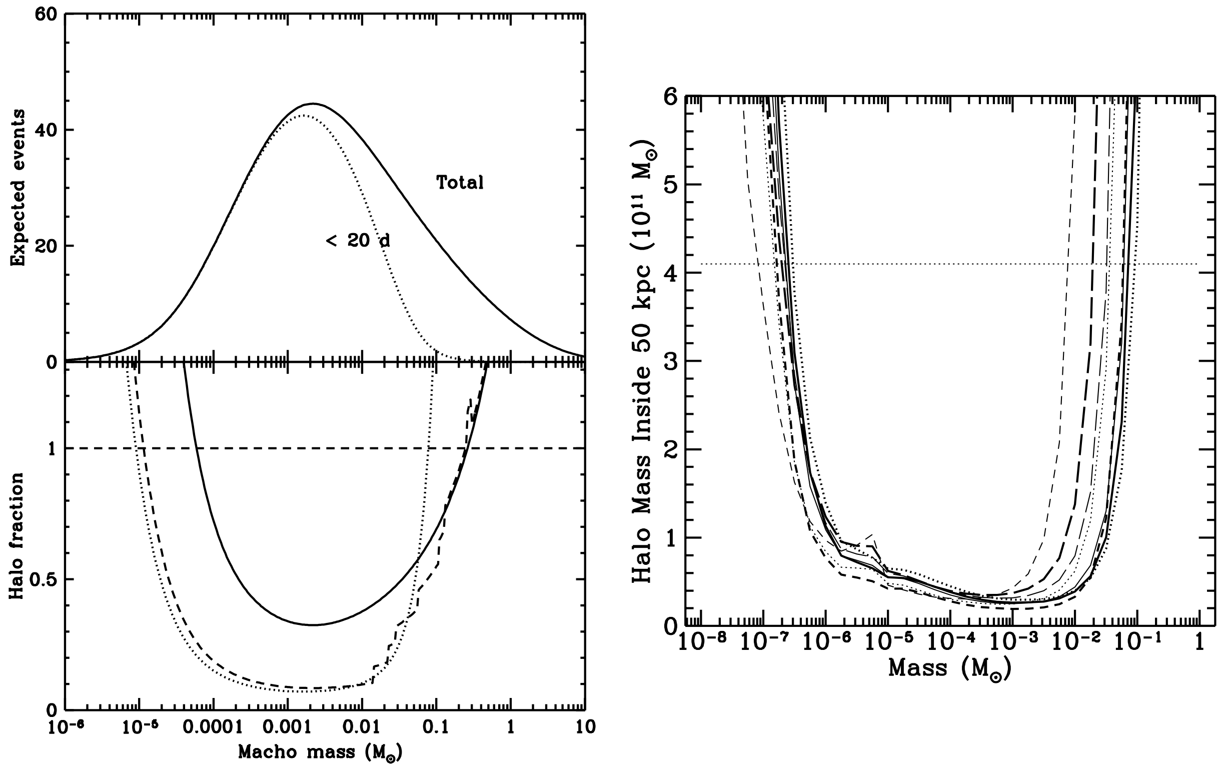

Figure 6: (a) The top left figure shows the expected number of microlensing events for the case when all lenses have the same mass m. The bottom left figure shows the restrictions of mass halo fraction which corresponds to the lenses with a such mass. The regions which are higher the drawn curves are excluded with the confidence probability 95%. The solid line is drawn using 8 observed events, dashed line is shown basing on the observational result that the events with the typical duration $\widehat{t} < 20$ days.
(b) The high limits (with confidence probability 95%) of the total halo mass which is formed by gravitational microlenses inside the region 50 *kpc* using results of processing observational data of the MACHO experiment for 8 different halo models. (Figure from the paper of Sutherland et al. [33]).

As it was noted earlier the typical duration of microlensing events depends essentially on the lens mass, therefore, if the lens mass is about $m \gtrsim 0.01\,\mathrm{M}_\odot$ then the most part of microlensing events have the typical duration about $\widehat{t} \gtrsim 10$ days, where the detection efficiency is high enough, but the rate of such events decreases with increasing the lens mass as $\propto m^{-0.5}$. For small lens masses $m < 0.001\,\mathrm{M}_\odot$ the estimated rate of such events is high enough, but the most part of events has the duration less than $\widehat{t} \sim 3$ days, where the detection efficiency of the MACHO experiment is small. As a result Sutherland et al. conclude that if the halo is formed by objects with mass $m \sim 2 \times 10^{-3}\,\mathrm{M}_\odot$ then maximal expected number of microlensing events in the MACHO is about ~ 45 [33].

From the absence of microlensing events with the short typical duration one can estimate the contribution of low mass lens into the halo mass. So, since the events with typical duration shorter than $\widehat{t} < 20$ days did not find in the MACHO experiment, Sutherland et al. concluded that the objects with the masses in the range from 6×10^{-5} up to $0.02\,\mathrm{M}_\odot$ contribute less than 20% halo mass with confidence probability 95% [33]. To increase a possibility of finding a low mass microlens (having short typical duration respectively), some regions of LMC were observed twice per night which allowed to get for a luminosity of any object in the regions a set from 4 points (two luminosities per night in each from two spectral bands). The microlensing events with the typical duration $\sim 0.3 - 3$ days did not find after corresponding processing observational data.

Analysing the data of the MACHO experiment, Sutherland et al. conclude that the lenses with the mass in the range from 10^{-6} to $0.02\,\mathrm{M}_\odot$ contribute less than 20% in the halo mass. In other words, such objects contribute less than $10^{11}\,\mathrm{M}_\odot$ in the halo mass inside 50 *kpc*, as shown in Fig. 6b [33].

In this case the estimated optical depth towards LMC (taking into account the detection efficiency of the MACHO experiment) is equal to [33]

$$\tau^{MACHO} = 2.9^{+1.4}_{-0.9} \times 10^{-7}. \qquad (11)$$

We note that earlier, basing on the data of the first year of observations (using 3 events), it was given the value for the optical depth τ^{MACHO} which is less approximately in three times [25].

Roulet and Mollerach note that it is necessary to accept critically the estimates for the optical depth τ from the expressions (10) and (11) because of the statistical data size of microlensing event candidates is not very large and the detection efficiency for the lens mass greater than $\sim M_\odot$ is small also [25]. On the other hand, probably some microlensing event candidates are not associated with real microlensing events actually. In this case the optical depth estimate must be less.

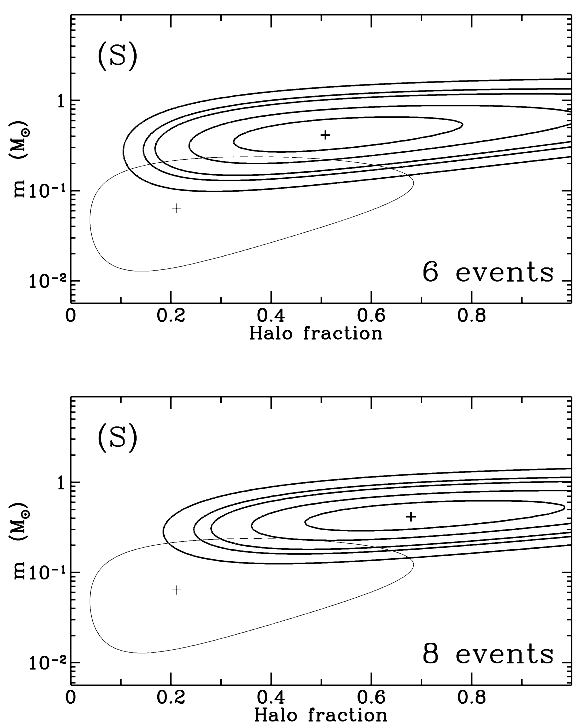

Figure 7: Thick lines show the contours (with the confidence probabilities 34,68,90,95,99% for the region inside the curves) for the lens mass and contributions of the lens masses in the halo mass for standard halo model for cases of 6 and 8 detected events. The thin line shows the contour corresponding to the confidence probability 90% and using the MACHO data of the first year of observations. (Figure from the paper of Sutherland et al. [33]).

If the standard light curve is observed then generally it is impossible to determine the distance between the lens and the observer. Since microlensing could be caused as well by stars in our Galaxy as the stars in LMC (as Sahu suggested [41]) or by halo objects. However, as Alcock et al. noted [42] that microlensing by known stars could lead to the detection about 1.1 events in the MACHO experiment and the optical depth $\tau_{\rm stars} \sim 0.5 \times 10^{-7}$ could be associated with the stars. So, the optical depth estimate exceeds the optical depth corresponding to the observable stars. Bennett et al. give the more conservative estimate of the optical depth of the halo $\tau_{\rm halo} = 2.1^{+1.1}_{-0.7} \times 10^{-7}$, obtained from the data analysis by the exclusion of event # 9 (since the lens corresponding to the case could be in LMC) and the exclusion of event # 10 (since corresponding background star could be variable) [39]. Sutherland et al. give the lens mass estimate, starting from the observable event durations. The variable $\widehat{t}$ depends on three unknown parameters: a lens mass, the distance between a lens and an observer, a transversal velocity. Therefore, the mass estimate has only a statistical significance and generally speaking depends only the halo model [33]. Using the maximal likelihood method one can determine the most probable interval of the lens mass масс линз $0.5^{+0.3}_{-0.2}\,{\rm M}_\odot$ for the standard halo model (Fig. 7). If the lenses were located in the halo, they would not be stars where the hydrogen burning is, since they would be detect in this case [43]. Thus, Sutherland et al think that the cold white dwarfs could be the most natural microlenses [33].

Although the formal significance of the experiment is high enough, Sutherland et al. do not assert that they found dark matter since several microlensing events suggested earlier are actually variable stars [33]. Zhao suggested [44] that the cause (which is artificial enough according to the opinion of Sutherland et al. [33]) of the observable optical depth could be associated with the hypothesis that there is a dwarf galaxy between LMC and an observer on the Earth, although the theoretical probability of this event is very small $\sim 1\%$.

2.4 Observations towards Galactic bulge

The extensive observations of the Galactic bulge are realized by the MACHO and OGLE collaborations which are yet mentioned. The French DUO (Disk Unseen Objects) collaboration carries out also the observations towards the bulge since 1993 in the La Silla Observatory of ESO in Chile using Schmidt telescope. As Ferlet reported, basing on communications of the group participants, the DUO collaboration got the light curves about 15 million stars and found several microlensing event candidates [45]. We shall discuss some results of observations of the OGLE and MACHO collaborations.

Observational results of the OGLE collaboration

Now it is known at least three collaborations which carried out the observations towards the Galactic bulge for the detection of the microlensing events. These collaborations found a lot of microlensing events (more than it was estimated before observations). The first group which started such observations is the OGLE collaboration which has been monitoring more than 10^6 stars since 1992. The most part of the observations of the collaboration is carried out in the I-band but some observations were performed in the V-band. The first microlensing event (if to put in order on time the maximal luminosity

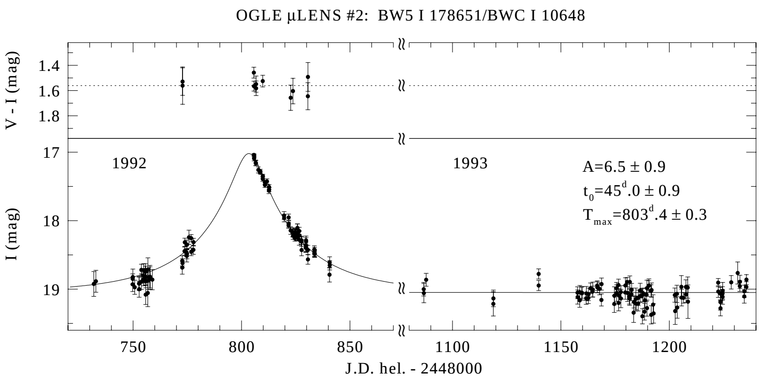

Figure 8: The example of the observed light curve corresponding to the point lens model: the event candidate OGLE #2 [46]. (Figure from the review of Paczynski [37]).

time of the observable light curves [37]) was event OGLE #10, which peaked on June 29, 1992 [46]. The event was found by the OGLE collaboration after six other events that were observed that summer. However, the event was extracted by a computer from observational data only on the spring of 1994. The OGLE collaboration discovered first event (OGLE #1) on September 22, 1993, but the event peaked on June 15, 1993, almost a year later than OGLE #10*. An example of the microlensing event (OGLE #2) is shown in Fig. 8. The microlensing event observations were in 1992. The event was in the overlap area of two separate fields, so it had a large number of measurements in the I-band: 93, 187, and 94 in the observing seasons 1993, 1994, and 1995 respectively. Paczynski pointed out on private communication of M. Szymanski, that stellar luminosity has been constant during these three year with average I-band magnitude of 19.07, 19.10 and 19.13 respectively, and standard deviation of individual measurements was 0.13, 0.10 and 0.09, respectively [37].

The most surprising result was the OGLE discovery that the optical depth is high as $3.3 \pm 1.2 \times 10^{-6}$ (based on 9 events) towards the Galactic bulge [30]. In the first theoretical paper Griest et al. [47] and Paczynski [48], where the optical depth towards the Galactic bulge was estimated, the effect of microlensing by the Galactic bulge stars was ignored. Kiraga and Paczynski supposed that the effect may be dominant for observations towards the Galactic bulge [49], however they still ignored the fact that there is a bar in the inner region of our Galaxy (a possibility of the bar existence was discussed by de Vaucouleurs [50] and Blitz and Spergel [51]). However, preliminary results of OGLE collaboration forced upon us the "rediscovery" of the Galactic bulge [46, 52, 53, 54, 55, 37]. So, Dwek et al. think that the microlensing searches are becoming a useful new tool for studies of the Galactic structure [57].

Besides the Galactic bar rediscovery the OGLE collaboration found the event corresponding to the light curve which is formed by a binary gravitational lens, when the source projection crosses the caustic curve being formed by the binary gravitational lens. Udalski et al presented the example of the light curve corresponding to the first binary lens (OGLE #7), the light curve is shown in Fig. 9 [60, 37]. The star was found to be constant in 1992,

* As Paczynski noted [37], basing on private communication of Alcock, the first event to be ever noticed by a human was MACHO # 1 - Will Sutherland saw it come out of a computer on Sunday 12, 1993.

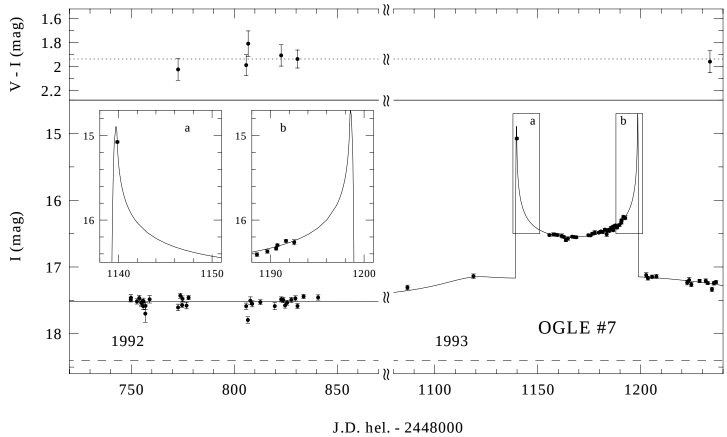

Figure 9: The possible example of the binary lens: the microlensing event candidate (OGLE #7) [60]. The regions of the two caustic crossing (a) and (b) are shown in the enlarged inserts. The MACHO collaboration has a few dozen additional points in two bands showing that the light curve variations are achromatic; three MACHO points correspond to the second caustic crossing (b) [58]. (Figure from the review of Paczynski [37]).

1994 and 1995. The average magnitude based on 32, 45 and 41 I-band measurements in these three observing seasons was 17.53, 17.52 и 17.54 respectively with the variance of single measurements being 0.07, 0.04 and 0.03 magnitudes, respectively [37]. Similar light curves were found also in the processing observational data of the MACHO collaboration [58]. The MACHO group also confirmed the presence of the second caustic crossing event near JD 2449200 and demonstrated that the light curve variation was achromatic.

The standard interpretation of the light curves as OGLE #7 is the intersection of the caustic curve which is formed by binary gravitational lens. However the caustic curve formation is also possible in the framework of the model of a transparent gravitational microlens being formed by a non-compact object [61, 18, 19].

Observational results of the MACHO collaboration

Alcock et al. informed about MACHO results of processing the observational data for first year observations towards the Galactic bulge [62]. The MACHO collaboration analysed 24 fields having about 12.6 million stars which are observed during 190 nights in 1993, and reported about detection of 45 microlensing event candidates having the durations from 4.5 up to 110 days*. The MACHO collaboration observed the bulge region having the galactic coordinates in the ranges $0^\circ < \ell < 7^\circ$ and $-2^\circ > b > -6^\circ$. Many of the event candidates had large enough signal/noise ratio and demonstrated remarkable examples of light curves. Using observations of 1.3 million stars from "the group of giants", typical distances and detection efficiencies which are well known, the MACHO collaboration found 13 microlensing event candidates in the region which had a square about the 13 squared degree on the celestial sphere and had the center with the galactic coordinates $\ell = 2.55^\circ$ and $b = -3.64^\circ$. Using the observational data the MACHO

* Sutherland et al. informed that the MACHO collaboration detected more than 100 microlensing events towards the bulge [33].

group calculated the estimate of the optical depth as $\tau^{MACHO} = 3.9^{+1.8}_{-1.2} \times 10^{-6}$. Preliminary results of the MACHO collaboration demonstrate that the optical depth grows with decreasing the coordinate $|b|$.

Interpretation of the observations

Since the observed optical depth towards the Galactic bulge is above the expectations from all these models, as already noted, Kiraga and Paczynski [49] and Paczynski et al. [53] suggested that the cause of these large rates could be the fact that the bulge is actually triaxial with larger axis making a small angle with respect to the line of sight [25]. In this case, the average distance between sources and lenses is larger, with corresponding increase of the associated Einstein radii and also the line of sight to a source goes through a larger number of lenses [25]. Therefore, the microlensing observations could make more precise model of our Galaxy. In addition, the velocities of stars in the bar would be smaller in the directions orthogonal to the major axis, helping to explain the event durations observed, i.e. $T \sim 10 - 50$ days, which would be too long for faint stars in a spherically symmetric model or axially symmetric bulge model [49], [25]. Predictions of the bar mass are consistent with the dynamical estimates $M_{bar} \simeq 2 \times 10^{10} M_{\odot}$, and the angle between the major axis and the line of sight is consistent with the Dwek et al. [57] inferred range $\alpha = 20^{\circ} \pm 10^{\circ}$ (see, for example, the review [25] and references in the paper).

Thus the observed optical depth could be caused both bar stars and faint stars of the disk or lenses which form dark matter. Moreover, as Roulet and Mollerach noted due to small velocity dispersions of disk constituents, the observed durations of microlensing events could correspond even to stars near the limit mass of brown dwarfs $0.08 M_{\odot}$ [25]. If lenses (and sources) belong to the bar one can expect that there is the rate asymmetry for positive and negative latitudes and the effect could be detected basing on the (observed) estimate of the rate in different fields. One may observe a magnitude offset between stars observed at opposite longitudes. with those at negative ℓ, which are further away being fainter [25]. Stanek et al. informed about the detection of the effect [52] by the OGLE collaboration. On the other hand, sources in the bar are more likely to be lensed if they are on the far side and hence there may also be an observed magnitude offset of the lens stars with respect to whole sample in given field [25]. The prediction was also confirmed in the OGLE experiment [52].

As Mollerach and Roulet noted, the complication in the interpretation of the bulge results is non-negligible fraction of the source stars actually belong to disk [64]. For example, Terndrup supposed that $\sim 15\%$ of the giant stars in Baade's Window are in the disk and this fraction may be even larger for main sequence stars [65, 25]. Moreover, in fields at larger longitudes and similar latitudes the number of disk stars is practically unaffected while that of bulge stars decreases significantly, making the effects of disk stars relatively more important in those fields [25]. The different distribution and the motion of source stars belonging to the disk affect the predictions about parameters of microlensing.

The main result of the observations towards the Galactic bulge is the fact that the observed optical depth is greater than the estimated optical depth. If the disk is not void then the rate of events being caused by disk stars is non-negligible and since in this case both a lens and a source are in the disk, the events have respectively long durations. The conclusion could explain the origin of events with durations (such as the long duration event which was found in MACHO experiment according to the communication of Alcock et al. [62]). If disk stars are microlensed by bar objects the optical depth could be greater for positive longitudes (in contrast to events when both a source and a lens are in the bar), features of the asymmetry would be decreased in the fields of observations.

Using results of observations towards the Galactic bulge, one can estimate the lens mass. As already noted, Mao and Paczynski concluded that it is necessary to analyse about 1000 events to get the reliable estimate of the lens mass in the case when the kind of a lens mass distribution is known but few parameters of the distribution are unknown [66]. According to Alcock et al., the preliminary analysis of the observations yields the lens mass estimate as 0.1–1. $M_\odot$ [62]. However, as Roulet and Mollerach noted, the lens mass function could be formed by two lens populations, i.e. the disk and bulge stars the populations have different lens mass distributions, and the interpretation of observational data would be more complicated in this case [25].

2.5 Results and unsolved problems

Let us cite well established results of microlensing searches and discuss the questions for which we have now different answers which do not contradict to the observational data [37]. Now it is generally recognized that the microlensing searches (when lenses are in our Galaxy or in nearby galaxies) are very important for solutions of different problems of astronomy and cosmology. As Paczynski noted, the most important is the consensus that the microlensing phenomenon has been discovered [37]. Now it is impossible to tell which part of the microlensing event candidates is actually connected with the effect since probably there are some variable stars among the event candidates, it could be stellar variability of unknown kind* .

1. Observed light curves are achromatic and their shapes are interpreted by simple theoretical expressions very well, however, there is not complete consent about "very well interpretation" since even for the event candidate MACHO # 1 the authors of the discovery proposed two fits. Dominik and Hirshfeld suggested that the event could be interpreted very well in the framework of the binary lens model [26, 27], but Gurevich et al. assumed that the microlensing event candidate could be caused by a non-compact microlens [11].

2. As expected, binary lenses have been detected and the observed rate of the events correspond to expected value.

3. As expected, the parallax effect has been detected.

4. Bennetti et al found that the spectrum of the only event monitored spectroscopically has been found to be constant throughout the intensity variations [67].

5. Since the observed optical depth is essentially greater than the estimated value, the independent (from considered early arguments) confirmation about the existence of the Galactic bar was done.

* The microlensing event candidates proposed early by the EROS collaboration (#1 and #2) and by the MACHO collaboration (#2 and #3) are considered now as the evidence of a stellar variability.

Now the following results are generally accepted [37]:

1. The optical depth towards the Galactic bulge is equal to $\sim 3 \times 10^{-6}$, so it is larger than the estimated value.

2. The optical depth towards the LMC is equal to $\sim 10^{-7}$, so, it is smaller than the estimated value. We recall that now the MACHO collaboration presented the estimate 3×10^{-7}, but the first MACHO estimate based on three events and the first EROS estimate based on two events coincided remarkably and were equal to 8×10^{-8}.

3. A lot of new interesting scientific results could be extracted from the giant data base which is collected during microlensing searches, thereby, as Schneider wrote, microlensing searches are "eldorado" for experts in stellar variability. New kinds of stellar variability were found already using microlensing observations, but probably the data base contain other interesting information and have the great scientific significance.

However there are different suggestions (which are not contradicted to the observational data) about the following issues [37]:

What is the location of objects which dominated microlensing towards the Galactic bulge?

Kiraga and Paczynski [49], Paczynski et al. [53], Zhao et al. [54, 55] suggested that most lenses are in the bulge, but Alcock et al. assumed that the most microlenses are in the Galactic disk [56].

Where are the most microlenses for searches towards LMC? The microlenses may be in the Galactic disk, Galactic halo, the LMC halo or in the LMC itself. Are the microlenses stellar mass objects or are they substellar brown dwarfs?

What fraction of microlensing events is caused by binary lenses?

What fraction of microlensing events is connected with binary stars?

Paczynski suggested that we shall have definite answers for some presented issues after some years and since the optical depth towards the Galactic bulge is essentially greater than the optical depth towards th LMC, we shall have more information about the lens distribution towards the Galactic bulge, however, probably, some problems in theoretical interpretation will appear after detection of new microlensing event candidates [37].

Conclusions

The main result of the microlensing searches is that the effect predicted theoretically has been confirmed. This is one of the most important astronomical discoveries.

When new observational data would be collected and the processing methods would be perfected, probably some microlensing event candidates lost their status, but perhaps new microlensing event candidates would be extracted among analysed observational data.

Perhaps the optical depth estimates would be changes especially for observations towards LMC since the recent MACHO estimate, given in 1996, is greater in several times than the estimate given by the EROS and MACHO collaborations in 1995. Moreover now one can not affirm about the coincidence of the estimates for the EROS and MACHO experiments. The lens mass estimates could be changed since early MACHO estimates (for searches towards LMC) are differed significantly from recent ones which we discussed above.

So, the general conclusion may be done. The very important astronomical phenomenon was discovered, but some quantitative parameters of microlensing will be specified in future.

Bibliographic remarks

One could find an introduction into the gravitational lens theory in a short report [69]. One could recommend the monograph [4] and the review [71] where there is a detailed description of the microlensing theory. In our presentation of the microlensing theory we also used essentially the recent comprehensive review [25] and the simple but very informative review [37]. Microlensing by non-compact objects is analysed in the papers [10, 11, 13, 15, 16, 18, 19]. The discussion of the "chromatic" features of microlensing is discussed in the paper [28].

A detailed description of the microlensing observations is in the book [4] and in the comprehensive review [71] or in the short review [70]. We used the papers [72, 73, 45, 31, 25] to describe in detail the EROS experiment. One could use these papers as useful bibliographic sources also. The results of the EROS experiment are described in the papers [72, 73, 45, 31, 25]. The MACHO results are presented in the comprehensive report of Alcock et al. [34] and in the paper [33] also. The brief description of the OGLE results is in the review of Paczynski [37].

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References

[1] Soldner J. G. Berliner Astron. Jahrbuch 1804, s. 161 (1804).

[2] Möller C. The Theory of Relativity (Oxford, Clarendon Press, 1952) [Translated into Russian (Moscow, Atomizdat, 1975)].

[3] Landau L. D., Lifshitz E. M. Teoriya polya (Field theory) (Moscow: Nauka, 1988).

[4] Zakharov A. F. Gravitatsionnie linzi i microlinzi (Gravitational Lenses and Microlenses) (Moscow: Janus, 1997).

[5] Liebes S. Phys. Rev. **133**, B835 (1964).

[6] Refsdal S. Month. Not. Royal Astron. Soc. **128**, 295 (1964)

[7] Refsdal S. Month. Not. Royal Astron. Soc. **128**, 307 (1964)

[8] Mukhanov V. F. Usp. Fiz. Nauk **135**, 133 (1981)

[9] Bliokh P.V., Minakov A.A. Gravitatsionnie linzi (Graviational Lenses) (Kiev, Naukova Dumka, 1989).

[10] Gurevich A.V., Zybin K.P. Phys. Lett. A. **208**, 276 (1995).

[11] Gurevich A.V., Zybin K.P., Sirota V.A. Phys. Lett. A **214**, 232 (1996).

[12] Gurevich A.V., Zybin K.P. Usp. Fiz. Nauk **165**, 723 (1995).

[13] Gurevich A.V., Zybin K.P., Sirota V.A. Usp. Fiz. Nauk **167**, 913 (1997).

[14] Zakharov A.F., Sazhin M.V. Pis'ma v Astron. Zhurn. **23**, 403 (1997).

[15] Zakharov A.F., Sazhin M.V. Pis'ma v Zh Eksp. Teor. Fiz. **63**, 894 (1996).

[16] Zakharov A.F., Sazhin M.V. Zh Eksp. Teor. Fiz. **110**, 1921 (1996).

[17] Zakharov A.F., Sazhin M.V. Phys. Atom. Nucl. **61**, 1220 (1998).

[18] Zakharov A.F. Phys. Lett. A **250**, 67 (1998).

[19] Zakharov A.F. Astron. Zhurn. **76**, 379 (1999).

[20] Byalko A.V. Astron. Zhurn. **46**, 998 (1969).

[21] Paczinsky B. Astrophys. J. **304**, 1 (1986).

[22] Griest K. Astrophys. J. **366**, 412 (1991).

[23] Alcock C. et al. Nat **365**, 621 (1993).

[24] Aubourg E. et al. Nat **365**, 623 (1993).

[25] Roulet E., Mollerach S. astro-ph/9603119 (1996).

[26] Dominik M., Hirshfeld A. C. Astron. and Astrophys. **289**, L31 (1994).

[27] Dominik M., Hirshfeld A.C. Preprint DO-TH 95/19, Dortmund (1995).

[28] Bogdanov M.B., Cherepashchuk A.M. Pis'ma v Astron. Zhurn. **21**, 570 (1995).

[29] Pratt M. et al. in IAU Symp. 173, eds C. Kochanek, J. Hewitt, Kluwer Academic, Dordrecht, (1995).

[30] Udalski A. et al. Acta Astron. **44**, 227 (1994).

[31] Magneville C. Annals of the New York Academy of Sciences, vol. 759, eds H. Böhringer, G. E. Morfill, J.E. Trümper, The New York Academy of Sciences, New York, p. 664, (1995).

[32] Palanque-Delabrouille N. et al. astro-ph/9611059 (1996).

[33] Sutherland W. et al. astro-ph/9611059 (1996).

[34] Alcock C. et al. astro-ph/9606165 (1996).

[35] Albrow M. in Proc. IAU Symp. 173, eds. C.S. Kochanek, J.N. Hewitt, Kluwer Academic Publishers, Dordrecht - Boston - London, p. 227 (1996).

[36] Ansari R. et al. Astron. and Astrophys. **299**, L21 (1995).

[37] Paczynski B. astro-ph/9604011 (1996).

[38] Ansari R. Nucl. Phys. B (Proc. Suppl.) **43** 108 (1995).

[39] Bennett D. et al. astro-ph/9606012 (1996).

[40] Alcock C. et al. astro-ph/9606165 (1996).

[41] Sahu K. Nat **370**, 275 (1994).

[42] Alcock C. et al. astro-ph/9603158 (1996).

[43] Flynn C., Gould A., Bahcall J. Astrophys. J. **466**, L55 (1996).

[44] Zhao H. astro-ph/9606166 (1996).

[45] Ferlet R. Annals of the New York Academy of Sciences, vol. 759, eds H. Böhringer, G. E. Morfill, J.E. Trümper, The New York Academy of Sciences, New York 56, (1995).

[46] Udalski A. et al. Acta Astron. **44**, 165 (1994).

[47] Griest K. et al. Astrophys. J. **372**, L79 (1991).

[48] Paczinsky B. Astrophys. J. **371**, L63 (1991).

[49] Kiraga M., Paczynski B. Astrophys. J. **430**, L101 (1994).

[50] De Vaucouleurs G. in IAU Symp. 20, The Galaxy and the Magellanic Clouds, F.J. Kerr & Rotgers (eds) (1964).

[51] Blitz L. & Spergel D.N. Astrophys. J. **370**, 205 (1991).

[52] Stanek K. et al. Astrophys. J. **429**, L73 (1994).

[53] Paczinsky B. et al. Astrophys. J. **435**, L113 (1994a).

[54] Zhao H., Rich R.M., Spergel D.N. Astrophys. J. **440**, L13 (1995).

[55] Zhao H., Spergel D.N., Rich R.M. astro-ph/9512065 (1995).

[56] Alcock C. et al. Astrophys. J. **445**, p. 133 (1995).

[57] Dwek E. et al. Astrophys. J. **445**, 716 (1995).

[58] Bennett D. et al. in AIP Conf. Proc. 336: Dark Matter, eds. S.S. Holt & C.L. Bennett (1995).

[59] Han C., Gould A. astro-ph/9504078 (1995).

[60] Udalski A. et al. Astrophys. J. **436**, L103 (1994).

[61] Ossipov D.L., Kuryan V.E. Phys. Lett. A **223**, 157 (1996).

[62] Alcock C. et al. astro-ph/9512146 (1995).

[63] Giudice G.F., Mollerach S. & Roulet E. Phys. Rev. **D50**, 2406 (1994).

[64] Mollerach S., Roulet E. Astrophys. J. **458**, L9 (1996).

[65] Terndrap D.M. AJ **96**, 884 (1988).

[66] Mao S., Paczynski B. astro-ph/9604002 (1996).

[67] Benetti S. et al. Astron. and Astrophys. **294**, L37 (1995).

[68] Schneider P. Preprint MPA 901 (1995).

[69] Zakharov A.F. Proc. of the XIX Internat. Workshop on High Energy Physics and Field Theory, Protvino, IHEP, eds V.A. Petrov, A.P. Samokhin, R.N. Rogalyov, p. 156, (1997).

[70] Zakharov A.F. Izvestia RAN (seria fizicheskaya) **62**, 1727 (1997).

[71] Zakharov A.F., Sazhin M. V. Usp. Fiz. Nauk **168**, 1041 (1998).

[72] Ansari R. in Particle Astrophysics, G. Fontaine & J. Tran Thanh Van, Editions Frontieres, p. 243, (1993).

[73] Ansari R. Annals of the New York Academy of Sciences, vol. 759, eds H. Böhringer, G. E. Morfill, J.E. Trümper, The New York Academy of Sciences, New York p. 608, (1995).